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Model Theoretic Methods in Finite Combinatorics

AMS-ASL Joint Special Session
January 5–8, 2009
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Martin Grohe
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Preface

From the very beginnings of *Model Theory*, applications to Algebra, Geometry and Number Theory accompanied its development and served as a source of inspiration. In contrast to this, applications to combinatorics emerged more recently and more slowly. Between 1975 and 1990 we saw the discovery of 0-1 laws for finite models of sentences of Predicate Logic by R. Fagin[1] and Y.V. Glebskii, D.I. Kogan, M.I. Liogonkii and V.A. Talanov[2], applications of Model Theory to generating functions by K.J. Compton[3] and to counting functions by C. Blatter and E. Specker[4]; and the emergence of algorithmic meta-theorems initiated by B. Courcelle[5]. The best known of these are the 0-1 laws which were, and still are, widely studied. The least known are the applications of Model Theory to combinatorial functions.

Methodologically, in the early applications of Model Theory to Algebra and Number Theory, elimination of quantifiers and the Compactness Theorem play crucial rôles, and the logic is always First Order Logic. In the applications to Combinatorics the logic often is Monadic Second Order Logic, and the tools are refinements of Ehrenfeucht-Fraïssé Games and of the Feferman-Vaught Theorem.

In recent years other very promising interactions between Model Theory and Combinatorics have been developed in areas such as extremal combinatorics and graph limits, graph polynomials, homomorphism functions and related counting functions, and discrete algorithms, touching the boundaries of computer science and statistical physics.

This volume highlights some of the main results, techniques, and research directions of the area. Topics covered in this volume include recent developments on 0-1 laws and their variations, counting functions defined by homomorphisms and graph polynomials and their relation to logic, recurrences and spectra, the logical complexity of graphs, algorithmic meta theorems based on logic, universal and homogeneous structures, and logical aspects of Ramsey theory. Most of the articles are expository and contain comprehensive surveys as well as new results on particular aspects of how to use Model Theoretic Methods in Combinatorics. They make

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[1] R. Fagin, *Probabilities on finite models*, J. Symb. Log. **41** (1976), no. 1, 50–58.

[2] Y.V. Glebskii, D.I. Kogan, M.I. Liogonkii, and V.A. Talanov, *Range and degree of realizability of formulas in the restricted predicate calculus*, Cybernetics **5** (1969), 142–154.

[3] K.J. Compton, *Applications of logic to finite combinatorics*, Ph.D. thesis, University of Wisconsin, 1980.

[4] C. Blatter and E. Specker, *Le nombre de structures finies d'une théorie à charactère fin*, Sciences Mathématiques, Fonds Nationale de la recherche Scientifique, Bruxelles (1981), 41–44, and *Recurrence relations for the number of labeled structures on a finite set*. In Logic and Machines: Decision Problems and Complexity (E. Börger, G. Hasenjaeger, and D. Rödding, eds.), Lecture Notes in Computer Science, vol. 171, Springer, 1984, pp. 43–61.

[5] B. Courcelle, *Graph rewriting: An algebraic and logic approach*, Handbook of Theoretical Computer Science (J. van Leeuwen, ed.), vol. 2, Elsevier Science Publishers, 1990, pp. 193–242.

the volume suitable to serve both as an introduction to current research in the field and as a reference text.

In spite of the title of this book, not all the articles deal with finite combinatorics in a strict sense. Two articles deal with countable homogeneous and countable universal structures, one article deals with partition properties of group actions, and two articles explore various aspects of Ramsey's Theorem. Still, they all share a model theoretic view and involve notions from finite Combinatorics.

Two applications of Model Theory to Combinatorics are, unfortunately, not covered in this volume, as they were published, or promised for publication elsewhere, before this volume came into being. G. Elek and B. Szegedy[6] used ultraproducts and Loeb measures to prove generalizations of Szemerédi's Regularity Lemma to hypergraphs. The Regularity Lemma plays a crucial rôle in the study of very large finite graphs. A. Razborov[7] used Model Theory to develop *flag algebras*, which allow one to derive results in extremal graph theory in a uniform way.

The volume is the outcome of the special session on

Model Theoretic Methods in Finite Combinatorics

that was held at the AMS-ASL Meeting of January 2009 in Washington D.C.

Most speakers of the special session, and a few other prominent researchers in the area, were invited to contribute to the volume. In the name of all the authors we would like to thank the referees for their careful reading of the contributions and their many suggestions which were incorporated into the final articles. Special thanks go to Christine Thivierge at the American Mathematical Society for patiently shepherding us through the publication process.

Martin Grohe
Johann Makowsky

[6]G. Elek and B. Szegedy, *Limits of hypergraphs, removal and regularity lemmas. A non-standard approach*, Available at: arXiv: 0705.2179, 2007.

[7]A. Razborov, *Flag algebras*, Journal of Symbolic Logic **72.4** (2007), 1239–1282.

This volume contains the proceedings of the AMS-ASL Special Session on Model Theoretic Methods in Finite Combinatorics, held January 5–8, 2009, in Washington, DC.

Over the last 20 years, various new connections between model theory and finite combinatorics emerged. The best known of these are in the area of 0-1 laws, but in recent years other very promising interactions between model theory and combinatorics have been developed in areas such as extremal combinatorics and graph limits, graph polynomials, homomorphism functions and related counting functions, and discrete algorithms, touching the boundaries of computer science and statistical physics.

This volume highlights some of the main results, techniques, and research directions of the area. Topics covered in this volume include recent developments on 0-1 laws and their variations, counting functions defined by homomorphisms and graph polynomials and their relation to logic, recurrences and spectra, the logical complexity of graphs, algorithmic meta theorems based on logic, universal and homogeneous structures, and logical aspects of Ramsey theory.

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