

# CONTEMPORARY MATHEMATICS

565

## Algebraic Groups and Quantum Groups

International Conference on Representation Theory  
of Algebraic Groups and Quantum Groups '10

August 2–6, 2010

Graduate School of Mathematics,  
Nagoya University, Nagoya, Japan

Susumu Ariki  
Hiraku Nakajima  
Yoshihisa Saito  
Ken-ichi Shinoda  
Toshiaki Shoji  
Toshiyuki Tanisaki  
Editors



American Mathematical Society

# Algebraic Groups and Quantum Groups

The International Conference, Graduate School of Mathematics, Nagoya University  
第10回名古屋国際数学コンファレンス

# Representation Theory of Algebraic Groups and Quantum Groups '10



*in front of the Toyoda Auditorium, Nagoya University, August 3, 2010*

# CONTEMPORARY MATHEMATICS

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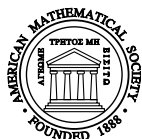
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American Mathematical Society  
Providence, Rhode Island

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## Preface

This volume is an outgrowth of the conference “Representation Theory of Algebraic Groups and Quantum Groups ’10” held at Nagoya University, Nagoya, Japan, during the period August 2-6, 2010, as the 10th International conference hosted by the Graduate School of Mathematics, Nagoya University.

This conference is the third one held in Japan under the same title. The first one was held at Sophia Univeristy in Tokyo, in August of 2001, and the second was held at Nagoya University, in June of 2006. This conference was planned as a continuation of those precedent ones. The purpose of these conferences is to focus on recent developments in the representation theory of algebraic groups and quantum groups. The conference also aims to bring young researchers a good opportunity for stimulating discussions with experts. During the conference, 22 lectures were delivered by invited speakers, which cover the topics such as modular representations of algebraic groups and finite groups of Lie type, representations of quantum groups and crystal bases, representations of affine Lie algebras and  $W$ -algebras, representations of Hecke algebras and Cherednik algebras, the theory of cluster algebras and its applications to the representation theory.

This volume contains fourteen articles contributed by invited speakers of the conference.

We are very grateful for the invited speakers for excellent lectures, for all the participants for their participation with great enthusiasm, and in particular for the contributors of the articles to this volume. We also thank the Grant-in-Aid for Scientific Research by the Japan Society for the Promotion of Science for the financial support.

Last but not least, we wish to thank Ms. Kazuko Kozaki and other secretaries, graduate students of the Graduate School of Mathematics, Nagoya Univeristy for their dedicated support of the conference.

Editors

Susumu Ariki(chair)  
Hiraku Nakajima  
Yoshihisa Saito  
Ken-ichi Shinoda  
Toshiaki Shoji  
Toshiyuki Tanisaki

November 2011





## Program

The International Conference  
Graduate School of Mathematics, Nagoya University

### Representation Theory of Algebraic Groups and Quantum Groups '10

- Period:** August 2 - 6, 2010
- Place:** Room 509, Building Science 1  
Graduate School of Mathematics  
Nagoya University  
Nagoya, Japan
- Organizers:** T. Shoji(chair), S. Ariki, H. Nakajima, Y. Saito,  
K. Shinoda, T. Tanisaki

#### August 2 (Mon)

- 10:00 - 11:00 S.-J. Kang (Seoul National Univ.)  
Crystal bases for quantum queer superalgebras
- 11:15 - 12:15 T. Arakawa (Nara Women's Univ.)  
Rationality and smoothness of  $W$ -algebras
- 13:45 - 14:45 D. Juteau (Univ. of Caen)  
Parity sheaves
- 15:00 - 16:00 M. Okado (Osaka Univ.)  
Stability in parabolic Lusztig  $q$ -analogues,  
one dimensional sums and fermionic formulas
- 16:30 - 17:30 T. Kuwabara (RIMS, Kyoto Univ.)  
Representation theory of the rational Cherednik algebra  
of type  $\mathbb{Z}/l\mathbb{Z}$  via micro local analysis

#### August 3 (Tue)

- 10:00 - 11:00 R. Bezrukavnikov (MIT)  
Canonical bases and modular representations
- 11:15 - 12:15 H. Nakajima (RIMS, Kyoto Univ.)  
Cluster algebras and quantum affine algebras

- 13:45 - 14:45 T. Nakanishi (Nagoya Univ.)  
Dilogarithm identities in conformal field theory and cluster algebras
- 15:00 - 16:00 N. Xi (Chinese Academy of Sciences)  
Some quotient algebras of affine Hecke algebras
- 16:30 - 17:30 Y. Saito (Univ. of Tokyo)  
Mirković-Vilonen polytopes and quiver construction of crystal basis in type  $A$

### August 4 (Wed)

- 10:00 - 11:00 V. Ginzburg (Univ. of Chicago)  
Isospectral commuting variety and the Harish-Chandra  $D$ -module
- 11:15 - 12:15 B. Feigin (Independent Univ. of Moscow)  
Representation theory of some version of quantum  $\mathfrak{gl}_\infty$
- 13:45 - 14:45 K. Nagao (Nagoya Univ.)  
Donaldson-Thomas theory and cluster algebras
- 15:00 - 16:00 M. Geck (Aberdeen Univ.)  
Generic representations of finite groups of Lie type
- 16:30 - 17:30 K. Wada (RIMS, Kyoto Univ.)  
On cyclotomic  $q$ -Schur algebras

### August 5 (Thu)

- 10:00 - 11:00 J. Brundan (Univ. of Oregon)  
Rational representations of  $GL(m|n)$  via Schur-Weyl duality
- 11:15 - 12:15 H. Miyachi (Nagoya Univ.)  
Hidden Hecke algebras and duality

### August 6 (Fri)

- 10:00 - 11:00 P. Fiebig (Erlangen Univ.)  
On the critical level representations of affine Kac-Moody algebras
- 11:15 - 12:15 A. Premet (Manchester Univ.)  
On 1-dimensional representations of quantized Slodowy slices
- 13:45 - 14:45 M. Kaneda (Osaka City Univ.)  
Some observation of the structure of  $F_*\mathcal{O}_{G/P}$
- 15:00 - 16:00 D. Sagaki (Tsukuba Univ.)  
Tensor product multiplicities for crystal bases of extremal weight modules over quantum infinite rank affine algebras of type  $B_\infty$ ,  $C_\infty$ , and  $D_\infty$

16:30 - 17:30 G.I. Lehrer (Univ. of Sydney)  
Equivariant  $K$ -theory of quantum homogeneous spaces  
and quantum affine spaces



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- 565 **Susumu Ariki, Hiraku Nakajima, Yoshihisa Saito, Ken-ichi Shinoda, Toshiaki Shoji, and Toshiyuki Tanisaki, Editors**, Algebraic Groups and Quantum Groups, 2012
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This volume contains the proceedings of the international conference on Representation Theory of Algebraic Groups and Quantum Groups '10, held August 2–6, 2010, at Nagoya University, Nagoya, Japan.

The survey articles and original papers contained in this volume offer a comprehensive view of current developments in the field. Among others reflecting recent trends, one central theme is research on representations in the affine case. In three articles, the authors study representations of  $W$ -algebras and affine Lie algebras at the critical level, and three other articles are related to crystals in the affine case, that is, Mirkovic–Vilonen polytopes for affine type  $A$  and Kerov–Kirillov–Reshetikhin type bijection for affine type  $E_6$ .

Other contributions cover a variety of topics such as modular representation theory of finite groups of Lie type, quantum queer super Lie algebras, Khovanov's arc algebra, Hecke algebras and cyclotomic  $q$ -Schur algebras,  $G_1T$ -Verma modules for reductive algebraic groups, equivariant  $K$ -theory of quantum vector bundles, and the cluster algebra.

This book is suitable for graduate students and researchers interested in geometric and combinatorial representation theory, and other related fields.

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