CONTEMPORARY MATHEMATICS

569

Real and Complex Singularities

XI International Workshop on
Real and Complex Singularities
July 26–30, 2010
Instituto de Ciências Matemáticas e de Computação
Universidade de São Paulo
São Carlos, SP, Brazil

Victor Goryunov
Kevin Houston
Roberta Wik-Atique
Editors

American Mathematical Society

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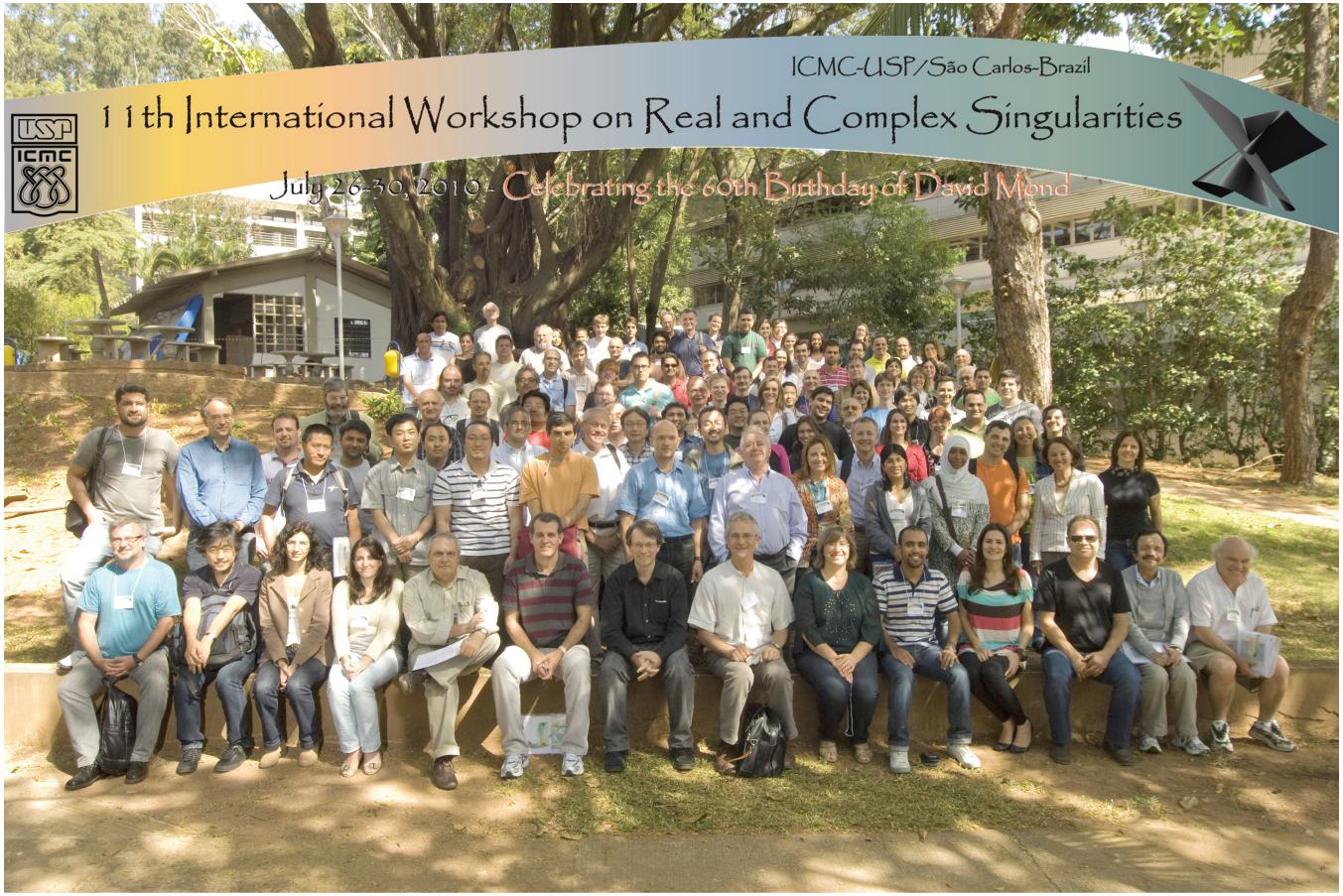

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This volume is dedicated to David Mond
on the occasion of his 60th birthday

Photo courtesy of Eleonore Faber.

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Preface

This volume is a collection of papers presented at the 11th Workshop on Real and Complex Singularities, São Carlos, Brazil, July 26-30, 2010. The meeting was a part of a highly successful series of biennial conferences organized by the Singularity Theory group at São Carlos, University of São Paulo, Brazil. It is the longest running sequence of international workshops in singularities, which have been providing an exceptional opportunity for both young researchers and recognised leaders working in the field to meet together in a very productive scientific atmosphere. In 2010 a total of 160 participants from 13 countries (Brazil, Canada, Denmark, France, Germany, Japan, Mexico, Norway, Poland, Portugal, Spain, UK, USA) came to the Workshop.

The meeting in 2010 had a special dedication – to David Mond's 60th birthday. David Mond, professor at the University of Warwick, United Kingdom, is one of the leading experts in the area and has inspired many young mathematicians. Many years ago he fell in love with South America. Nowadays David has very strong connections with the Singularity group at Sao Carlos.

The main subject of singularity theory is the geometry and topology of spaces and maps defined by polynomials or analytic equations which are not regular. The theory uses techniques from several branches of mathematics and contributes to the development of rather distant fields, including algebraic geometry, knot theory, optics, computer vision, and many others. The possibility of application in a large number of different areas is one of the reasons of the success of singularity theory.

This book reflects the high level of the conference. It discusses the most recent results and applications of singularity theory and shows promising directions for future research in the field. Therefore, it will be an excellent reference for experienced researchers and an ideal introduction for younger, such as PhD students and post-docs, not only for current results but also for the variety of methods and techniques used in singularity theory.

The volume covers pure singularity theory (invariants, classification theory, Milnor fibres) and applications to other areas (singularities in topology and differential geometry, algebraic geometry and bifurcation theory). In particular, the reader will find here papers on plane curve singularities, metric theory of singularities, symplectic singularities, cobordisms of maps, Goursat distributions, sections of analytic varieties, Vassiliev type invariants, projections of hypersurfaces, properties of the Jacobian ideal.

We thank members of the Scientific Committee: Ragnar Buchweitz, Francisco Castro Jiménez, James Damon, Washington L. Marar, Walter Neumann, Juan José Nuño Ballesteros, Marcio Soares and Duco van Straten. We are also thankful to members of the Organizing Committee: Alexandre Fernandes, Nivaldo Grulha Jr,

Regilene Oliveira, Marcelo Saia, João Tomazella and Catiana Casonatto. We would like to express our gratitude to all others who work hard for the success of the meeting.

The workshop was funded by Brazilian funding bodies Fapesp, CNPq, CAPES, USP and INCTMat and the Japanese funding body JSPS, whose support we gratefully acknowledge.

We thank the referees for their diligent work in refereeing all the papers in this volume. We thank the staff members of the American Mathematical Society involved with the preparation of this book, and all those who have contributed in whatever way to these proceedings.

Victor Goryunov
Kevin Houston
Roberta Wik-Atique

David Mond

To celebrate the 60th birthday of David Mond the 2010 Real and Complex Singularities conference, a meeting held biennually in São Carlos, Brazil, was dedicated to him. David has been an organizer and proceedings editor for earlier Real and Complex Singularities conferences. Furthermore, having two of his PhD students based in São Carlos means that David has strong connections with the singularity group there and that the conference was a great opportunity to honour him.

David had not originally intended to become an academic. After completing his undergraduate degree in Mathematics and Philosophy at St. Catherine's College, Oxford, 1971, he was inspired by the work of Dutch furniture maker Gerrit Rietveld and initially pursued the ambition of making furniture himself. In 1973 with an Oxford friend he visited South America intending to travel through Venezuela and Colombia to Peru. Their journey came to a halt in Bogotá, Colombia, where David remained in the end for eight years.

It was in Colombia that David returned to mathematics, taking a Master's degree in Mathematics in the Universidad Nacional de Colombia, and later working as a mathematics instructor at Universidad de los Andes, and Universidad Nacional, both in Bogotá. His first five, essentially expository, papers appeared in Colombian mathematical journals and hence were written in Spanish – a language in which, along with Portuguese, he is fluent. After finishing the MSc in Mathematics at the Universidad Nacional, he moved in 1979 to the UK to begin a PhD at the University of Liverpool under the supervision of C.T.C. Wall FRS. His PhD, submitted in 1982, was entitled *The Classification of Germs of Maps from Surfaces to 3-space, with Applications to the Differential Geometry of Immersions.* From the late 1960s there had been much progress in the classification of singularities, particularly for right equivalence following V.I. Arnold's classifications. What was novel and significant about David's thesis was that it concerned the $\mathcal{A}$-equivalence (also known as right-left equivalence) for germs from $\mathbb{R}^2$ to $\mathbb{R}^3$. The stable maps in this case were classified by Whitney back in the 50s, David was classifying simple maps and those with $\mathcal{A}_e$-codimension less than or equal to 4. In the late 70s and early 80s such a classification required doing by hand long, tedious calculations involving tangent spaces, although David insists that it is often through long tedious calculations that one builds up an understanding of what is going on. Aspects of the classification were applied in differential geometry, for example to study the tangent developable of a space curve.

After his PhD, David had a brief research visit at the IHES in France and returned to Colombia to work again at the Universidad Nacional. In 1985 he moved to the University of Warwick in the UK. Of particular note during this time is the publication *Some remarks on the geometry and classification of germs*

of maps from surfaces to 3*-space* in Topology 26 (1987) 361-383. It is here that David starts a thread that passes through much of his work by studying complex analytic maps, in this case investigating how invariants of map germs from $\mathbb{C}^2$ to $\mathbb{C}^3$ control the determinacy of the germ. It is in this paper that the (still unresolved) Mond conjecture first begins to take form: the $\mathcal{A}_e$-codimension of a map germ from $(\mathbb{C}^n, 0)$ to $(\mathbb{C}^{n+1}, 0)$ is less than or equal to the rank of the vanishing homology for the image of a local stabilization of the map, with equality if the map is quasihomogeneous. Various Mond students - including this author - have attempted to prove this statement during their PhD studies.

Also at this time David had his first PhD student, Washington 'Ton' Marar from Brazil, with whom he further developed the ideas in the Topology paper to show the multiple point spaces for finitely $\mathcal{A}$-determined corank 1 maps from $\mathbb{C}^n$ to $\mathbb{C}^p$, $n < p$, were in fact isolated complete intersection singularities (or zero-dimensional). His interest in classification continued with his second student, Diana Ratcliffe, producing a thesis on classifying multi-germs from $\mathbb{R}^2$ and $\mathbb{R}^3$. She later became his post-doctoral assistant and produced a computer program that relieved the drudgery from the extensive calculations involved in classifications. David remarked in one seminar that her program could do a calculation in seconds that he had spent weeks doing during his postgraduate days. A visit to Holland at this time led to a collaboration with Ruud Pellikaan, and later to a series of papers with Duco van Straten.

Other students followed in the early 90s with Thomas Cooper and this author both completing in 1994. By this time David had collaborated with other notable Singularity Theorists: Terry Gaffney, on the germs of maps from the plane to the plane; Jim Damon (in their paper for Inventiones Mathematicae) on a result that related $\mathcal{A}_e-$codimension and the rank of the vanishing homology for maps with $n \geq p$ (which added evidence to the Mond conjecture); Victor Goryunov, on a powerful spectral sequence that allowed one to calculate the rational cohomology of the image of a finite map.

David continued his association with São Carlos, co-supervising Roberta Wik-Atique with Maria Ruas, again on classification of singularities. However, following the importance of free divisors in the paper co-authored with Damon, he began to study these objects in more detail. A hypersurface D is a free divisor if the module of ambient vector fields which are tangent to D at its smooth points is freely generated. This property has significant consequences for the topology of the singularity, and, as the Inventiones paper showed, for the topology of its non-linear sections, because it implies the conservation of a certain singular multiplicity. This interest eventually produced a string of papers singly and co-authored with people such as Francisco Castro-Jiménez, Luis Narváez-Macarro, Martin Holland, Francisco Calderón Moreno, Ragnar-Olaf Buchweitz, Michel Granger, Alicia Nieto-Reyes, Mathias Schulze, Ignacio de Gregorio (another PhD student and later postdoctoral assistant) and Christian Sevenheck. In this string of papers, the important concept of linear free divisor is introduced. (This is a free divisor where the basis is formed by linear vector fields.) The importance partly stems from the connection to quiver representations as shown in the paper with Buchweitz. They also have connection with Frobenius manifolds since for a large class of examples, the Gauss-Manin system associated to a generic linear section of the Milnor fibre of a linear free divisor is a Frobenius manifold.

Recent students have been Paul Cadman (2010) and Ayşe Altintas (2011). The latter was one of David's students present at the conference in his honour. The conference was a great success and during the outing to a nearby lake the attendees were treated to a performance of David playing the flute. At the conference dinner friends, colleagues and ex-students took turns to pay tribute and sing the praises of a man they are proud to call their friend and teacher.

Kevin Houston
Leeds, UK, 2011.

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