CONTEMPORARY MATHEMATICS

577

Multi-Scale and High-Contrast PDE: From Modelling, to Mathematical Analysis, to Inversion

Conference on Multi-Scale and High-Contrast PDE: From Modelling, to Mathematical Analysis, to Inversion June 28–July 1, 2011 University of Oxford, United Kingdom

> Habib Ammari Yves Capdeboscq Hyeonbae Kang Editors



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Preface

The mathematical analysis of PDE modelling materials, or tissues, presenting multiple scales have been an active area of research for more than 40 years. The study of the corresponding imaging, or reconstruction, problem is a more recent one. If the material parameters of the PDE present high contrast ratio, then the solution to the PDE becomes particularly challenging to analyze, or compute. Similar difficulties occur in time dependent equations in high frequency regimes. On the other hand, high frequency regimes, or very contrasted materials, were considered first in imaging, as well-differentiated areas are, at first sight, simpler to locate by ad-hoc methods. Over the last decade the analysis of the inversion problem at moderate frequencies, the rigorous derivation of asymptotics at high frequencies, and the regularity properties of solutions of elliptic PDE in highly heterogeneous media have received a lot of attention.

The focus of this volume is on recent progress towards a complete understanding of the direct problem with high contrast or high frequencies, and unified approaches to the inverse and imaging problems for both small and large contrast or frequencies.

The volume includes contributions on the inverse problem, both on its analysis and on numerical reconstructions. It offers the reader a good overview of current research and direction for further pursuit on multiscale problems, both in PDE and in signal processing, and in the analysis of the equations or the computation of their solutions. Finally, a special attention is devoted to new models and problems coming from physics leading to innovative imaging methods.

The tremendous success of the workshop was only possible due to the enthusiastic participation of wonderful speakers and authors of this volume. We are thankful to all of them. We also acknowledge with gratitude the generous support from the Engineering and Physical Sciences Research Council, the Oxford Centre for Nonlinear PDE, the Oxford Centre for Collaborative Applied Mathematics, the National Research Foundation of Korea, and the European Research Council Project MULTIMOD. We would also like to thank the Mathematical Institute of the University of Oxford.

Habib Ammari, Yves Capdeboscq, and Hyeonbae Kang

This volume contains the proceedings of the conference "Multi-Scale and High-Contrast PDE: From Modelling, to Mathematical Analysis, to Inversion", held June 28–July 1, 2011, at the University of Oxford.

The mathematical analysis of PDE modelling materials, or tissues, presenting multiple scales has been an active area of research for more than 40 years. The study of the corresponding imaging, or reconstruction, problem is a more recent one. If the material parameters of the PDE present high contrast ratio, then the solution to the PDE becomes particularly challenging to analyze, or compute. Similar difficulties occur in time dependent equations in high frequency regimes. Over the last decade the analysis of the inversion problem at moderate frequencies, the rigorous derivation of asymptotics at high frequencies, and the regularity properties of solutions of elliptic PDE in highly heterogeneous media have received a lot of attention.

The focus of this volume is on recent progress towards a complete understanding of the direct problem with high contrast or high frequencies, and unified approaches to the inverse and imaging problems for both small and large contrast or frequencies. The volume also includes contributions on the inverse problem, both on its analysis and on numerical reconstructions. It offers the reader a good overview of current research and direction for further pursuit on multiscale problems, both in PDE and in signal processing, and in the analysis of the equations or the computation of their solutions. Special attention is devoted to new models and problems coming from physics leading to innovative imaging methods.



