CONTEMPORARY MATHEMATICS

596

Advances in Ultrametric Analysis

12th International Conference
p-adic Functional Analysis
July 2–6, 2012
University of Manitoba, Winnipeg, Canada

Khodr Shamseddine
Editor

American Mathematical Society

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Group photo from the 12th International Conference on *p*-adic Functional Analysis, held July 2–6, 2012, at the University of Manitoba, Winnipeg, Canada

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10 9 8 7 6 5 4 3 2 1 18 17 16 15 14 13

Contents

Preface vii

Affine invariant measures in Levi-Civita vector spaces and the Erdös obtuse angle theorem
Martin Berz and Sebastian Troncoso 1

Some old and new results on zeros of the derivative of a p-adic meromorphic function
Jean-Paul Bézivin, Kamal Boussaf, and Alain Escassut 23

Survey on p-adic meromorphic functions $f'P'(f), g'P'(g)$ sharing a small function and additional properties
Kamal Boussaf, Alain Escassut, and Jacqueline Ojeda 31

The p-adic q-distributions
Bertin Diarra 45

Morphisms between ultrametric Banach algebras and maximal ideals of finite codimension
Alain Escassut and Nicolas Maïnetti 63

Survey on branched values and exceptional values for p-adic meromorphic functions
Alain Escassut and Jacqueline Ojeda 73

Grobman-Hartman theorems for diffeomorphisms of Banach spaces over valued fields
Helge Glöckner 79

Integral representations of continuous linear maps on p-adic spaces of continuous functions
Athanasios K. Katsaras 103

Subfields of valued, complete fields
Hans A. Keller 123

On some classes of non-Archimedean operator algebras
Anatoly N. Kochubei 133

Some identities and congruences for Stirling numbers of the second kind
Hamadoun Maïga and Fana Tangara 149

Non-measurable sets in the Levi-Civita field
Héctor M. Moreno 163

Partial fractional differentiability
Enno Nagel 179

A generalized Keller space over a field with a valuation of rank $\alpha > \omega$
Herminia Ochsenius and Elena Olivos 205

A comprehensive survey of non-archimedean analysis in Banach spaces over fields with an infinite rank valuation
Herminia Ochsenius and Elena Olivos 215

All proper multiplications on the completion of a totally ordered abelian group
Elena Olivos and Wim H. Schikhof 237

The Grothendieck approximation theory in non-Archimedean functional analysis
Cristina Perez-Garcia 243

A brief survey of the study of power series and analytic functions on the Levi-Civita fields
Khodr Shamseddine 269

On non-Archimedean generalized power series spaces
Wiesław Śliwa 281

Preface

This volume contains papers based on lectures given at the 12th International Conference on p-adic Functional Analysis which was held at the University of Manitoba from July 2 to July 6, 2012. The conference followed the tradition of eleven earlier meetings which took place in Laredo (Spain, 1990), Santiago (Chile, 1992), Clermont-Ferrand (France, 1994), Nijmegen (The Netherlands, 1996), Poznan (Poland, 1998), Ioannina (Greece, 2000), Nijmegen (The Netherlands, 2002), Clermont-Ferrand (France, 2004), Concepción (Chile, 2006), East Lansing (USA, 2008), and Clermont-Ferrand (France, 2010).

The 2012 conference and its proceedings, published in this special volume of Contemporary Mathematics, are dedicated to our colleagues who have retired since our previous meeting in 2010: Professors Bertin Diarra, Alain Escassut, Athanasios Katsaras, Hans Keller, and Herminia Ochsenius who have continuously enriched this series of conferences and the related proceedings with their original research and wise guidance. We wish them the best and we look forward to their continued contributions in the future.

The articles included in this book feature recent developments in various areas of non-Archimedean Analysis: Branched values and zeros of the derivative of a p-adic meromorphic function, partial fractional differentiability, morphisms between ultrametric Banach algebras of continuous functions and maximal ideals of finite dimension, p-adic q-distributions, Grobman-Hartman theorems for diffeomorphisms of Banach spaces over valued fields, integral representations of continuous linear maps on p-adic spaces of continuous functions, non-Archimedean operator algebras, generalized Keller spaces over valued fields, proper multiplications on the completion of a totally ordered abelian group, generalized power series spaces, and measure theory on Levi-Civita fields. The book also includes four survey papers: one paper on p-adic meromorphic functions sharing a small function (by Kamal Boussaf, Alain Escassut and Jacqueline Ojeda), one on Banach spaces over fields with an infinite rank valuation (by Herminia Ochsenius and Elena Olivos), one on the Grothendieck approximation theory in non-Archimedean Functional Analysis (by Cristina Perez-Garcia), and my paper on power series and analytic functions on Levi-Civita fields.

For their guidance and help while organizing the conference and during the refereeing process, I am very grateful to the members of the conference's Scientific Committee: José Aguayo, Jesus Araujo, Gilles Christol, Bertin Diarra, Alain Escassut, Athanasios Katsaras, Herminia Ochsenius, Cristina Perez-Garcia, and Wim Schikhof. I am also grateful to the University of Manitoba and the Winnipeg Institute for Theoretical Physics for their financial and logistical support for the

conference. For their contributions to this book of the proceedings of the conference, I would like to thank all the authors and referees of the papers therein as well as the American Mathematical Society. Finally, for her instrumental support and encouragement for me, and for caring for our two angels Adam and Latifa when I was very busy working for the conference, I thank my wife (and colleague) Samar Safi-Harb.

The next meeting in the series will be organized by Helge Glöckner at the University of Paderborn (Germany) in August 2014. We will look forward to that.

Khodr Shamseddine

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Through a combination of new research articles and survey papers, this book provides the reader with an overview of current developments and techniques in non-Archimedean analysis as well as a broad knowledge of some of the sub-areas of this exciting and fast-developing research area.

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