

CONTEMPORARY MATHEMATICS

599

Geometric Analysis, Mathematical Relativity, and Nonlinear Partial Differential Equations

Southeast Geometry Seminars
Emory University, Georgia Institute of Technology,
University of Alabama, Birmingham,
and the University of Tennessee
2009–2011

Mohammad Ghomi
Junfang Li
John McCuan
Vladimir Oliker
Fernando Schwartz
Gilbert Weinstein
Editors



American Mathematical Society

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Preface

This volume presents the proceedings of the Southeast Geometry Seminar for the meetings that took place between the Fall of 2009 and the Fall of 2011. The one-day events were held bi-annually, and their location was rotated among four hosting institutions: Emory University, Georgia Institute of Technology, University of Alabama Birmingham, and the University of Tennessee.

The conference series has been running continuously since the spring of 2002. During this period, the meetings grew from a dozen participants to about 30 participants during the last meetings. The conference attracts researchers and students working in Geometric Analysis and related fields. The original rationale for the series was that since geometric analysis research groups in the region are typically small, short and frequent meetings which are accessible by road travel would foster collaboration and help these groups grow and attract students.

Topics for the conference series include geometric analysis and related fields, in particular, nonlinear partial differential equations, general relativity, and geometric topology. Geometric analysis has seen several major developments in the last two decades. Among these are a number of important breakthroughs, such as the global stability of Minkowski spacetime and Perelman's work on Hamilton's Ricci flow. These developments have generated a lot of activity, pushing the discipline forward. The field also has numerous applications outside geometry in many diverse fields, such as general relativity, quantum field theory, capillarity, topology, optics, microbiology, and image processing. The area holds the promise of many more exciting developments in the next few years.

The Southeast Geometry Seminar received financial support from the National Science Foundation (grant DMS-0940878). The organizational assistance provided by the hosting institutions, and in particular by the grant administrator, Ms. Cheryl Logan at the University of Alabama, was also essential for its success.

Fernando Schwartz
Knoxville, TN
March 2013

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This volume presents the proceedings of the Southeast Geometry Seminar for the meetings that took place bi-annually between the fall of 2009 and the fall of 2011, at Emory University, Georgia Institute of Technology, University of Alabama Birmingham, and the University of Tennessee. Talks at the seminar are devoted to various aspects of geometric analysis and related fields, in particular, nonlinear partial differential equations, general relativity, and geometric topology.

Articles in this volume cover the following topics: a new set of axioms for General Relativity, CR manifolds, the Mañé Conjecture, minimal surfaces, maximal measures, pendant drops, the Funk-Radon-Helgason method, ADM-mass and capacity, and extrinsic curvature in metric spaces.

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