CONTEMPORARY MATHEMATICS

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Centre de Recherches Mathématiques Proceedings

Tropical and Non-Archimedean Geometry

Bellairs Workshop in Number Theory
Tropical and Non-Archimedean Geometry
May 6–13, 2011
Bellairs Research Institute,
Holetown, Barbados

CRM

Omid Amini
Matthew Baker
Xander Faber
Editors

American Mathematical Society
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2010 *Mathematics Subject Classification.* Primary 14T05, 14G22.

Library of Congress Cataloging-in-Publication Data

Bellairs Workshop in Number Theory (2011 : Holetown, Barbados)
Tropical and non-Archimedean geometry : Bellairs Workshop in Number Theory, May 6–13, 2011, Bellairs Research Institute, Holetown, Barbados / Omid Amini, Matthew Baker, Xander Faber, editors.
pages cm – (Contemporary Mathematics ; volume 605)(Centre de recherches mathématiques proceedings)
Includes bibliographical references.
ISBN 978-1-4704-1021-6(alk. paper)
1. Tropical geometry–Congresses. 2. Geometry, Analytic–Congresses. I. Amini, Omid, 1980– editor of compilation. II. Baker, Matthew, 1973–editor of compilation. III. Faber, Xander, editor of compilation. IV. Title.

QA582.B45 2011
516.3′5–dc23   2013027062

Contemporary Mathematics ISSN: 0271-4132 (print); ISSN: 1098-3627 (online)

DOI: http://dx.doi.org/10.1090/conm/605

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Preface

1. Overview of the workshop

The Bellairs Workshop in Number Theory — held annually at the Bellairs Research Institute in Holetown, Barbados — is a gathering of mathematicians allied to learn a current topic of research interest. The venue is rather primitive: dormitory style housing, picnic table seating, and small chalk boards. And yet the location and enthusiasm of the participants make this one of the most coveted invitations available to number theorists.

The 2011 Workshop was focused on emerging connections between two subjects: tropical geometry and non-Archimedean geometry. The principal speaker was Matthew Baker, who gave a series of four introductory lectures covering different aspects of tropical geometry and connections to non-Archimedean geometry with a particular emphasis on the case of curves. This included an introduction to tropicalization and polyhedral structures, Payne's theorem connecting tropicalization and analytification, the structure theory of Berkovich curves, specialization of linear series from algebraic to tropical curves, and applications to the algorithmic problem of finding implicit equations for parametrically defined algebraic curves.

These lectures were supplemented and enhanced with complementary talks by various participants:

- Antoine Ducros gave an introductory talk on Berkovich analytic spaces.
- Joe Rabinoff gave a talk on admissible formal schemes, tropical integral models, and the Sturmfels – Tevelev multiplicity formula.
- Amaury Thuillier presented an overview of potential theory on Berkovich curves.
- Mihran Papikian talked about non-Archimedean uniformization and monodromy pairing.
- Diane Maclagan treated polyhedral structures on tropicalizations arising from Gröbner bases.
- Sergey Norin spoke about the combinatorics of linear series on tropical curves and reduced divisors.
- Antoine Chambert-Loir gave a survey of recent progress in Diophantine Geometry over function fields, including recent work of Gubler on the Bogomolov conjecture (in which Berkovich spaces and their tropicalizations play a significant role).

The articles in this volume by Chambert-Loir, Maclagan, and Papikian are based, at least in part, on their lectures at the Bellairs workshop. A portion of Baker's lectures correspond to the article by Baker, Payne and Rabinoff in this volume, and also to a longer version of the same article [1]. The other articles selected for inclusion in this volume represent other facets of current research and illuminate one or more connections between tropical geometry, non-Archimedean geometry, toric geometry, algebraic graph theory, and algorithmic aspects of systems of polynomial equations.

All articles in this volume were anonymously refereed and conform to high standards for research and/or exposition.

2. Tropical Geometry from a Non-Archimedean Viewpoint

We now present a more detailed overview of the topics of the Bellairs Workshop, as well as the contents of this Proceedings volume.

Let K be a complete non-Archimedean field with nontrivial absolute value $|\cdot|$ and valuation $\mathrm{val}(\cdot) = -\log|\cdot|$, for example $\mathbb{Q}_p$ or the field of formal Laurent series $\mathbb{C}(\!(t)\!)$.

The absolute value on K extends uniquely to any finite extension, and hence also to the completion of an algebraic closure of K. We will abuse notation and write $|\cdot|$ and $\mathrm{val}(\cdot)$ for the extended valuation as well. We denote the value group of K by $G = \mathrm{val}(K^\times)$.

Newton polygons. If $f \in K[X^{\pm 1}]$ is a Laurent polynomial, the classical theory of Newton polygons allows one to determine the valuations of the roots of f (counted with multiplicities) from the valuations of the coefficients of f. By definition, if $f(X) = \sum a_n X^n$, the Newton polygon $\mathrm{NP}(f)$ is the lower convex hull of the set

$$S_f = \{(n, \mathrm{val}(a_n)) : a_n \neq 0\} \subset \mathbb{R}^2.$$

The classical theorem of the Newton polygon asserts that there exists $x \in \overline{K}^\times$ with $\mathrm{val}(x) = r$ and $f(x) = 0$ if and only if $-r$ is a slope of $\mathrm{NP}(f)$, and the number of roots of valuation r is precisely the length of the horizontal projection of the segment with slope $-r$.

Example 2.1. • Consider the polynomial $f(X) = X^2 - (p+1)X + p \in \mathbb{Q}_p[X]$, where we assume that the absolute value is normalized so that $|p| = 1/p$, or equivalently, $\mathrm{val}(p) = 1$. Then the roots of f are $x = p$ and $x = 1$, which have valuation 1 and 0, respectively. See Figure 1(a).

• Now consider $f(X) = X^2 - p \in \mathbb{Q}_p[X]$. Evidently both roots of f have valuation $\frac{1}{2}$. See Figure 1(b).

Tropical hypersurfaces. Let us further assume that K is algebraically closed; e.g., $K = \mathbb{C}_p$ or $\mathbb{C}\{t\}$, the completion of the field of formal Puiseux series. Let $f \in K[X_1^{\pm 1}, \ldots, X_n^{\pm 1}]$. We use multi-index notation to write $f = \sum a_I X^I$ with $a_I \in K$ and $I \in \mathbb{Z}^n$. Let $\mathbb{T} = (K^\times)^n$ be the n-dimensional torus.

For $x \in \mathbb{T}$, set

$$\mathrm{trop}(x) = (\mathrm{val}(x_1), \ldots, \mathrm{val}(x_n)), \quad x = (x_1, \ldots, x_n).$$

For $w \in \mathbb{R}^n$, define $\varphi_w : \mathbb{Z}^n \to \mathbb{R} \cup \{\infty\}$ by

$$\varphi_w(I) = I \cdot w + \mathrm{val}(a_I).$$

[1] Nonarchimedean geometry, tropicalization, and metrics on curves. Available at arXiv:1104.0320

FIGURE 1. (a) One segment of the Newton polygon of f has x-length 1 and slope -1, while the other has x-length 1 and slope 0. (b) The Newton polygon of f has a single segment of x-length 2 and slope $-\frac{1}{2}$.

By a simple observation, if $w = \operatorname{trop}(x)$, then $\varphi_w(I) = \operatorname{val}(a_I x^I)$. Moreover, if $f(x) = 0$, then there must be at least two monomial terms $a_I X^I$ and $a_J X^J$ such that $\varphi_w(I) = \varphi_w(J)$ and such that φ_w achieves its minimum there.

In accordance with this observation, we define $\operatorname{Trop}(f)$ to be the union of all $w \in \mathbb{R}^n$ for which φ_w achieves its minimum on at least two points of $\mathbb{Z}^n$.

Note that $\operatorname{Trop}(f)$ depends only on the valuations of the coefficients a_I. It is not hard to show that $\operatorname{Trop}(f)$ is the corner locus (i.e., locus of nondifferentiability) of the piecewise linear function $\mathbb{R}^n \to \mathbb{R}$ given by

$$w \mapsto \min_{a_I \neq 0} \varphi_w(I).$$

A fundamental theorem of Kapranov asserts that $\operatorname{Trop}(f)$ coincides with the topological closure of $\operatorname{trop}(V(f))$ in $\mathbb{R}^n$, where we write $V(f)$ for the hypersurface in $\mathbb{T}$ cut out by the Laurent polynomial f.

More precisely, one has $\operatorname{trop}(V(f)) = \operatorname{Trop}(f) \cap G^n$.

EXAMPLE 2.2. Let $f(X, Y) = X + Y + p \in \mathbb{Q}_p[X, Y]$. Then $\operatorname{Trop}(f)$ is the corner locus of $(x, y) \mapsto \min\{x, y, 1\}$. See Figure 2.

For $n = 1$ and $f \in K[X]$, we have

$$\operatorname{Trop}(f) = \{w \in \mathbb{R} : -w \text{ is a slope of } \operatorname{NP}(f)\},$$

where $\operatorname{NP}(f)$ denotes the Newton polygon of f. Kapranov's theorem implies that as sets, we have

$$\{-\text{slopes of } \operatorname{NP}(f)\} = \{\text{valuations of roots of } f\}. \tag{1}$$

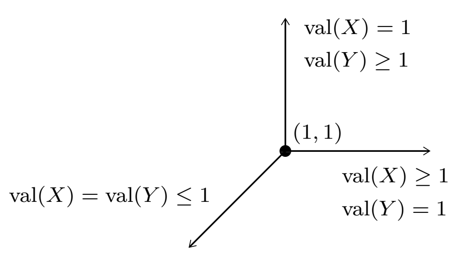

FIGURE 2. An illustration of $\operatorname{Trop}(f)$ for $f(X, Y) = X + Y + p$. The edge labels show the monomial valuations for f.

This raises the question, "What about multiplicities?" The roots of f have a natural notion of multiplicity attached to them. Below we will see a natural way to realize $\mathrm{Trop}(f)$ as a polyhedral complex with multiplicities attached to the maximal faces.

Newton complex of a tropical hypersurface. The Newton polytope of f, denoted $\mathrm{NP}(f)$, is defined to be the convex hull of $\{I \in \mathbb{Z}^n : a_I \neq 0\}$. The Newton complex $\mathrm{New}(f)$, sometimes called the Newton subdivision of $\mathrm{NP}(f)$, is the polyhedral structure on $\mathrm{NP}(f)$ defined as follows: $\{I_1, \ldots, I_r\}$ is a face if and only if there exists $w \in \mathbb{R}^n$ such that $\{I_1, \ldots, I_r\} = \{I : \varphi_w(I) \text{ is minimal}\}$.

This definition determines a polyhedral structure on $\mathrm{Trop}(f)$: w, w' belong to the same open face if and only if

$$\{I : \varphi_w(I) \text{ is minimal}\} = \{I : \varphi_{w'}(I) \text{ is minimal}\}.$$

Generalizing the usual definition of the Newton polygon (which is the case $n = 1$), one shows that $\mathrm{New}(f)$ is the projection to $\mathbb{R}^n$ of the lower convex hull of the set $\{(I, \mathrm{val}(a_I))\} \subset \mathbb{Z}^n \times \mathbb{R}$ with the induced polyhedral structure.

The tropicalization of f is dual to $\mathrm{New}(f)$. More precisely, the poset of faces of $\mathrm{Trop}(f)$ is canonically dual to the poset of positive-dimensional faces of $\mathrm{New}(f)$, and each face of $\mathrm{Trop}(f)$ is orthogonal to its dual face in $\mathrm{New}(f)$.

The correct way to generalize (1) as an equality of multi-sets to the case where $n > 1$ is to introduce an appropriate notion of tropical multiplicities: The weight or tropical multiplicity of a maximal face σ of $\mathrm{Trop}(f)$ is the lattice length of the corresponding edge of the Newton complex.

EXAMPLE 2.3. The polynomial $f(X,Y) = X^2Y + XY^2 + (1/p)XY + X + Y \in \mathbb{Q}_p[X,Y]$ determines a smooth affine algebraic curve in $\mathbb{T}$ whose smooth projective completion has genus 1. See Figure 3.

Tropical varieties. Let $I \subset K[X_1^{\pm 1}, \ldots, X_n^{\pm 1}]$ be an ideal, and let $X = V(I) \subset \mathbb{T}$ be the associated closed subvariety of $\mathbb{T}$. One defines

$$\mathrm{Trop}(X) = \bigcap_{f \in I} \mathrm{Trop}(f).$$

A collection of polynomials $f_1, \ldots, f_r$ is a tropical basis for I if

$$\mathrm{Trop}\big(V(I)\big) = \bigcap_{i=1}^{r} \mathrm{Trop}(f_i).$$

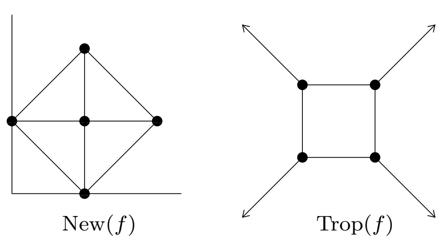

FIGURE 3. The Newton complex and tropicalization of f. Each edge of $\mathrm{Trop}(f)$ has tropical multiplicity 1.

It is important to note that a generating set for I is not in general a tropical basis. However, there is a tropical analogue of Hilbert's Basis Theorem: there always exists a finite tropical basis for I. As a consequence, $\mathrm{Trop}(X)$ can be given the structure of a G-rational polyhedral complex (but not in a canonical way).

One has the topological equality $\mathrm{Trop}(X) = \overline{\mathrm{trop}(X)}$, generalizing Kapranov's theorem. In addition, there is an important result due to Bieri and Groves which asserts that if $X \subset \mathbb{T}$ is a d-dimensional irreducible variety, then $\mathrm{Trop}(X)$ is the support of a pure d-dimensional connected G-rational polyhedral complex.

The article of Maclagan contains a discussion of how one defines a polyhedral structure on $\mathrm{Trop}(X)$ and includes a proof of the tropical analogue of Hilbert's Basis Theorem.

The article of Osserman and Rabinoff describes a generalization of the theory of Newton polygons to higher dimensions. More specifically, suppose that $f_1, \ldots, f_n \in K[X_1^{\pm 1}, \ldots, X_n^{\pm 1}]$ and assume that $X = V(f_1) \cap \cdots \cap V(f_n)$ is zero-dimensional. Then if w is an isolated point of $\mathrm{Trop}(X)$, the number of $x \in X(\bar{K})$ with $\mathrm{trop}(x) = w$ (counted with multiplicities) can be described combinatorially in terms of the Newton complexes of the f_i. Specifically, let $C_i(w)$ be the cell of $\mathrm{New}(f_i)$ corresponding to the monomials in f_i having minimal valuation at w. Then the number of $x \in X(\bar{K})$ with $\mathrm{trop}(x) = w$, counted with multiplicities, is equal to the mixed volume $MV\big(C_1(w), \ldots, C_n(w)\big)$. This can be viewed as a generalization of (1) (as an equality of multi-sets) to the case of n (Laurent) polynomials in n variables. It should be thought of as asserting that the number of classical intersection points of $V(f_1), \ldots, V(f_n)$ tropicalizing to a given point w is equal to the tropical intersection multiplicity $MV\big(C_1(w), \ldots, C_n(w)\big)$ of $\mathrm{Trop}(f_1), \ldots, \mathrm{Trop}(f_n)$ at w.

Algorithmic applications. There is an extensive literature on algorithmic aspects of tropical geometry and connections with computational algebra and algebraic geometry. The article of Phillipson and Rojas in this volume studies one particular aspect of this: the question of the maximal number of nondegenerate roots of a sparse system of n polynomial equations in n variables over a local field. This is linked to the construction of combinatorially constrained tropical varieties with maximally many intersections. Phillipson and Rojas also discuss links to some fundamental questions in complexity theory.

Non-Archimedean coamoebae. What we have been calling $\mathrm{Trop}(X)$ is the non-Archimedean version of an amoeba, defined as the image in $\mathbb{R}^n$ of a subvariety X of a complex torus $(\mathbb{C}^*)^n$ under the map $z \mapsto \log|z|$. A coamoeba is the image of X under the argument map $z \mapsto \arg(z)$, and the phase tropical variety is the closure of the image of X under the pair of maps, tropicalization and argument. The article of Nisse and Sottile in these proceedings studies an analogous construction of non-Archimedean coamoebae and phase tropical varieties.

3. Tropicalization and analytification

Berkovich analytic curves. Let K be a complete non-Archimedean field, which for simplicity we assume to be nontrivially valued and algebraically closed. Let X be an affine K-variety. As a topological space, one defines the Berkovich analytic space X^{an} to be the set of multiplicative seminorms on the coordinate ring

$K[X]$ extending the given absolute value on K, endowed with the weakest topology such that the map $|\cdot|_x \mapsto |f|_x$ is continuous for all $f \in K[X]$.

One can show that $X(K)$ is dense in X^{an}, and that X^{an} is a locally compact, Hausdorff, and locally path-connected space. One can also globalize the construction of X^{an} to separated schemes of finite type over K (as well as to rather general kinds of rigid analytic spaces and formal schemes).

The article of Baker – Payne – Rabinoff in this volume gives a detailed description of X^{an} in the special case where X is a smooth projective algebraic curve. In particular, the analytic version of the semistable reduction theorem asserts that there exists a finite set V of points of X^{an} such that $X^{\mathrm{an}}\backslash V$ is the disjoint union of infinitely many open balls and finitely many open annuli. Such a subset V is called a semistable vertex set for X^{an}. Associated to each semistable vertex set V one has a canonical subset $\Sigma = \Sigma(X^{\mathrm{an}}, V)$ of X^{an} called the skeleton of X^{an} with respect to V. It is a finite and connected metric graph and there is a canonical retraction map $\tau\colon X^{\mathrm{an}} \to \Sigma$. (In fact, a theorem of Berkovich asserts that Σ is a deformation retract of X^{an}.) The inverse limit of $\Sigma(X^{\mathrm{an}}, V)$ over all semistable vertex sets V is canonically homeomorphic to X^{an}.

Extended tropicalization and Payne's theorem. Let M be a free abelian group of rank n, let $N_{\mathbb{R}} = \mathrm{Hom}(M, \mathbb{R})$, and let $\mathbb{T} = \mathrm{Spec}\, K[M]$ be the algebraic torus over K whose character lattice is M. If X is a closed subvariety of $\mathbb{T}$, the tropicalization map $(x_1, \ldots, x_n) \mapsto \big(\mathrm{val}(x_1), \ldots, \mathrm{val}(x_n)\big)$ can be described more intrinsically as the map $\mathrm{trop}\colon X(K) \to N_{\mathbb{R}}$ defined by sending x to $\big\{u \mapsto \mathrm{val}\big(\chi^u(x)\big)\big\} \in N_{\mathbb{R}}$, where χ^u is the character of $\mathbb{T}$ corresponding to $u \in M$. This extends naturally to a continuous map $\mathrm{trop}\colon X^{\mathrm{an}} \to N_{\mathbb{R}}$, and we have $\mathrm{Trop}(X) = \mathrm{trop}(X^{\mathrm{an}})$ (no closure required!).

These considerations can be extended from tori to toric varieties. Let Δ be a strongly convex rational polyhedral fan in $N_{\mathbb{R}}$ and let Y_Δ be the corresponding toric variety.Then there is a natural "partial compactification" $N_{\mathbb{R}}(\Delta)$ of $N_{\mathbb{R}}$ and a natural extended tropicalization map $\mathrm{trop}\colon X^{\mathrm{an}} \to N_{\mathbb{R}}(\Delta)$ for any closed subvariety X of Y_Δ. Set-theoretically, $N_{\mathbb{R}}(\Delta)$ is the disjoint union of the tropicalizations of all torus orbits in Y_Δ; the topology is defined in such a way that the natural map $Y_\Delta(K) \to N_{\mathbb{R}}(\Delta)$ extends to a continuous, proper, and surjective map $\mathrm{trop}\colon Y_\Delta^{\mathrm{an}} \to N_{\mathbb{R}}(\Delta)$ (see the article of Osserman-Rabinoff for further details; such partial compactifications play an important role in tropical intersection theory.)

If X is an arbitrary quasi-projective variety over K, the embeddings of X into (quasi-projective) toric varieties Y_Δ form a directed system in a natural way, and a theorem of Payne asserts that the natural map from X^{an} to the inverse limit of the extended tropicalizations $\mathrm{trop}(X^{\mathrm{an}})$ over all such embeddings is a homeomorphism.

Tropicalization of subvarieties of abelian varieties. In addition to tropicalizing subvarieties of tori (and more generally toric varieties), one can also tropicalize subvarieties of abelian varieties using the non-Archimedean uniformization theory of Mumford and Raynaud.

The article of Papikian in this volume gives an introduction to non-Archimedean uniformization theory for curves and Abelian varieties.

The article of Chambert-Loir provides an overview of Gubler's work applying tropicalizations of subvarieties of abelian varieties to the function field analogue of the Bogomolov Conjecture in arithmetic geometry.

Tropical curves and their Jacobians. Let X be a smooth proper curve over K and let $\Sigma = \Sigma(X^{\mathrm{an}}, V)$ be a skeleton of X^{an}. The corresponding retraction map $\tau\colon X^{\mathrm{an}} \to \Sigma$ is analogous in many ways to a tropicalization map (it depends on the choice of a semistable vertex set V rather than the choice of a toric embedding of X). The skeleton Σ is a metric graph which can be profitably thought of as an "abstract tropicalization" of X.

Thinking along such lines, one can view an arbitrary metric graph Γ as an "abstract tropical curve"; this point of view was introduced and developed by Kontsevich, Soibelman, Mikhalkin, and others without reference to Berkovich spaces, but it also fits in very naturally with Berkovich's theory. In this way, many classical facts about algebraic curves and their Jacobians have tropical analogues.

For example, if Γ is a metric graph, one can define a tropical rational function on Γ to be a continuous piecewise linear function with integer slopes. One can define the divisor of such a function to be

$$\mathrm{Div}(f) = \sum_{p\in\Gamma} \sigma_p(f)(p),$$

where $\sigma_p(f)$ is the sum of the outgoing slopes of f at p. The divisors obtained in this way are called principal and form a subgroup $\mathrm{Prin}(\Gamma)$ of the group $\mathrm{Div}^0(\Gamma)$ of degree-zero divisors on Γ. The tropical Jacobian $\mathrm{Jac}(\Gamma)$ is defined to be the quotient group $\mathrm{Div}^0(\Gamma)/\mathrm{Prin}(\Gamma)$. As shown by Mikhalkin and Zharkov, $\mathrm{Jac}(\Gamma)$ is canonically isomorphic to the g-dimensional real torus $H_1(\Gamma,\mathbb{R})/H_1(\Gamma,\mathbb{Z})$, where $g = \dim_{\mathbb{R}} H_1(\Gamma,\mathbb{R})$ is the genus of Γ.

The tropical Jacobian is equipped with a canonical quadratic form which one thinks of as the tropical analogue of a principal polarization. There is a tropical version of the Torelli theorem due to Caporaso and Viviani which explains precisely to what extent the metric graph Γ is determined by its principally polarized tropical Jacobian. The article of Viviani in this volume describes the moduli space of (vertex-weighted) abstract tropical curves of genus g and its relation to the moduli space of principally polarized tropical abelian varietes, with applications to studying compactifications of the classical Torelli map $M_g \to A_g$.

The equivalence relation on $\mathrm{Div}(\Gamma)$ defined by declaring that $D \sim D'$ if and only if $D - D' \in \mathrm{Prin}(\Gamma)$ is closely related to the classical combinatorial theory of chip-firing, as first pointed out by Baker and Norine in their paper establishing a Riemann – Roch theorem for graphs. (There is a corresponding Riemann – Roch theorem for tropical curves, proved independently by Gathmann – Kerber and Mikhalkin – Zharkov.) The graph-theoretic analogue of the tropical Jacobian is often called the sandpile group or critical group of a graph. The article of Perkinson–Perlman–Wilmes in this volume details a number of algebraic and combinatorial aspects of the sandpile group.

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Over the past decade, it has become apparent that tropical geometry and non-Archimedean geometry should be studied in tandem; each subject has a great deal to say about the other.

This volume is a collection of articles dedicated to one or both of these disciplines. Some of the articles are based, at least in part, on the authors' lectures at the 2011 Bellairs Workshop in Number Theory, held from May 6-13, 2011, at the Bellairs Research Institute, Holetown, Barbados.

Lecture topics covered in this volume include polyhedral structures on tropical varieties, the structure theory of non-Archimedean curves (algebraic, analytic, tropical, and formal), uniformization theory for non-Archimedean curves and abelian varieties, and applications to Diophantine geometry. Additional articles selected for inclusion in this volume represent other facets of current research and illuminate connections between tropical geometry, non-Archimedean geometry, toric geometry, algebraic graph theory, and algorithmic aspects of systems of polynomial equations.

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ISBN 978-1-4704-1021-6

9 781470 410216

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