CONTEMPORARY MATHEMATICS

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Tropical and Idempotent Mathematics and Applications

International Workshop
Tropical and Idempotent Mathematics
August 26–31, 2012
Independent University, Moscow, Russia

G. L. Litvinov
S. N. Sergeev
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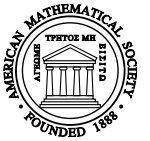

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Preface

Tropical mathematics is a new branch of mathematics, which rapidly progressed over the last few decades. Its linear-algebraic foundations were laid in the 1960s by Cuninghame-Green and Vorobyev. In the 1980s and 1990s the Russian group led by Academician Maslov and the French Max-plus Working group independently observed that certain problems in discrete optimization, optimal control and Hamilton-Jacobi-Bellman PDE are linear over the *max-plus semiring*, i.e., the set of real numbers completed with $-\infty$ and equipped with "addition" $a \oplus b := \max(a, b)$ and "multiplication" $a \otimes b := a+b$. This tropical linearity gave rise to a new systematic approach to such problems. Moreover, it was observed that one can exploit the following passage to the limit: $\lim_{h \to +0} h \log(e^{a/h} + e^{b/h}) = \max(a, b)$. This passage, now often called the *Litvinov-Maslov dequantization*, allows one to transfer the results from the realm of "traditional" mathematics over the real and complex field to the realm of tropical mathematics. The Litvinov-Maslov dequantization, being closely related to the dequantization procedures of theoretical physics, gives rise to a correspondence principle between the "traditional" and tropical mathematics, which can be seen as our main paradigm.

In fact, it can be argued that the origins of modern tropical mathematics are much more diverse. In particular, the recent success of *tropical algebraic geometry* (associated, in particular, with the names of Mikhalkin, Passare, Sturmfels and Viro) was inspired by the seminal work of Gelfand, Kapranov and Zelevinsky who introduced the notion of amoeba in algebraic geometry. However, the classical problems of algebraic geometry were formulated already in the 19th century. Some of the most recent applications of tropical linearity in numerical linear algebra (bounds on the roots of polynomials), developed by Gaubert, Sharify and others, are closely related to the classical works of Ostrowski and Polya. In the same vein, the origins of *tropical convexity* can be found in the works of Isbell on injective metric spaces, and there are notable contributions from abstract convex analysis (by Rubinov, Singer and others). On top of that, the tropical mathematics has been enriched by various real-life applications in railway scheduling, discrete event systems, phylogenetic analysis, static analysis of programs, electroenergetic networks, chemical reactions, and others. Some of these applications do appear in this volume.

A number of international events entirely devoted to Tropical Mathematics have been organized. The first international conference on Idempotency was held in Bristol in 1994 and a subsequent collection of papers with the same name was edited by Gunawardena. Three subsequent conferences on Tropical and Idempotent Mathematics were held in Vienna (2003), Moscow (2007) and Montreal (2009). Let us also mention a conference on Tropical Geometry and Integrable Systems in

Glasgow (2011), Tropical Semester at MSRI (2009) and a number of mini-symposia on tropical linear algebra in the framework of ILAS and SIAM conferences.

This volume of the Contemporary Mathematics series is also entirely devoted to Tropical/Idempotent Mathematics. It is a collection of papers submitted by participants of the conference "Tropical-12" held in August 2012 at the Independent University of Moscow. This collection contains some papers on the topics of tropical linear algebra, tropical convexity and idempotent analysis that have become well established, within the tropical mathematics. Note that some of these papers are devoted to extensions of the tropical semiring (like the symmetrized semiring or the supertropical semiring) or such "quasitropical" semirings as the max-min semiring. In some of the papers, optimization over tropically convex sets is considered. There are interesting contributions related to tropical geometry (tropical Plücker functions) and applications in chemistry (tropical equilibria of chemical reactions) and electroenergetic networks, as well as more abstract and general issues of semiring theory (MV-semirings and MV-algebras) and applications of semirings in single-agent and multi-agent dynamics. In this volume, see a new paper of V. P. Maslov on the foundations of classical thermodynamics.

We are grateful to our friends from the Russian-French Laboratory "J.-V. Poncelet", the Moscow Centre for Continuous Mathematical Education, the Independent University of Moscow, the Institute for Information Transmission Problems of RAS, and the Institute for Control Problems of RAS, for their help and support in organizing the conference "Tropical-12" in Moscow. We thank the American Mathematical Society and personally Sergei Gelfand and Christine Thivierge for their support. Finally, we are indebted to all the researchers of our Tropical/Idempotent community who contributed towards the creation of this volume.

Sergeĭ Sergeev,
Birmingham (UK), 2013.

In Memory of Grigory Litvinov

One of the co-editors of this volume, Professor G. L. Litvinov, passed away on July 12, 2013. We recall some basic facts from his mathematical biography.

G. L. Litvinov was born on May 16, 1944 in Moscow. A student of P. K. Rashevskiĭ and F. A. Berezin, he graduated and received his Ph.D. from M.V. Lomonosov Moscow State University, Department of Mechanics and Mathematics, in 1969. His initial research interests concerned Representations of Lie Groups and Topological Group Algebras. His early results in Representation Theory and Functional Analysis include, for instance, a classification of all completely irreducible representations of nilpotent Lie groups and a solution to A. Grothendieck's trace problem ("problème de biunivocité").

In the 1970s and 1980s, G. L. Litvinov worked at the All-Union State Institute of Statistics and Technology (as project manager), at the All-Union Research Institute of Documents and Archives, and at the All-Union State Patent Library (as head of the Department of Computer Science). He examined new methods of constructing rational and spline approximations, as well as various statistical models and methods. In particular, while studying the rational approximations of functions he discovered and formulated an effect of autocorrection. Meanwhile he continued his research in pure mathematics (locally convex spaces, representations of Lie groups and Lie algebras, the Plancherel formula, hypergroups and hypergroup algebras).

In the early 1990s G. L. Litvinov joined the group of Academician V.P. Maslov who developed Idempotent Analysis. The revolutionary appeal of this new branch

of mathematics was expressed in the Litvinov-Maslov correspondence principle, according to which:

There is a (heuristic) correspondence between important, useful and interesting constructions and results over the field of real (or complex) numbers (or the semiring of nonnegative numbers) and similar constructions and results over idempotent semirings in the spirit of the correspondence principle in Quantum Mechanics.

This paradigm of Idempotent Mathematics was accompanied by the Idempotent Superposition Principle in Hamilton-Jacobi PDE and Optimal Control, as well as observations of deep analogies between the Fourier transform and the Legendre-Fenchel transform, or between the Feynman's path integral and variational principles of Classical Mechanics.

In the late 1990s and 2000s, G. L. Litvinov (with V. P. Maslov and G. B. Shpiz) was developing Idempotent Functional Analysis (tensor products of idempotent spaces, nuclears semimodules and kernel theorems, linear functionals on idempotent spaces, Hahn-Banach theorem). His other contributions to Idempotent/Tropical Mathematics include Idempotent Interval Analysis and Universal Algorithms of Linear Algebra (with A. N. Sobolevskiĭ, A. Ya. Rodionov and A. V. Tchourkin).

Grigory Lazarevich supervised and influenced many mathematicians of younger generations, and he is survived by the Tropical/Idempotent community. We are grateful to him for all of his effort and his time devoted to the organization of three international conferences on Tropical/Idempotent Mathematics: Vienna (2003), Moscow (2007) and Moscow (2012), and five collections of articles which he edited including the present volume of *Contemporary Mathematics*.

In what follows, we have collected some warm responses and condolences from colleagues and friends of Grigory Lazarevich (Grisha) Litvinov.

Viktor P. Maslov (Moscow, Russia):

"Litvinov was a very special friend of mine. I would say, we "found" (or, rather, "discovered") each other. Grisha told me not long ago that it was him who discovered me first, after my talk on idempotent (tropical) mathematics. I objected to Grisha that I had discovered him before that talk, meaning his works on groupoids. He and Vitya Ivriy had impressed me by the breadth and depth of their mathematical talent. I was also impressed by Grisha's ability to accumulate students and colleagues (like G. Shpiz, A. Rodionov, S. Sergeev, and others).

Grisha was not only a talented mathematician, but also a deep thinker and a remarkable master of language. I felt some emptiness when I could not talk to him. The fact, that this disaster happened to him at the height of his scientific career and plans, was not surprising to me. I know what it means to lose both wife and mother in a couple of years, and the sudden death of Grisha's wife was the worst of all. Realizing that only a child could help him recover from this, I proposed that Grisha should also invite his housekeeper's daughter to live with him. Grisha rejoiced as he occupied himself with that child. I am also grateful to his housekeeper Galya who took care of his mother and then, as he became ill, of Grisha himself.

However, one also has to feel responsible for health. Litvinov, who suffered from diabetes, could not endure the strict diet imposed on him. Moreover, Grisha followed an ancient merchant's tradition of treating his guests with tea and biscuits. In particular, I adopted his habit of drinking tea from a small plate. When I "taste" tea like this, I feel the presence of his spirit. I hope that Grisha Litvinov's favourite

student G. Shpiz, who works in Russia, as well as his young colleagues A. Rodionov and S. Sergeev, who work abroad, will continue his scientific life."

Colleagues from the A.A. Kharkevich Institute for Information Transmission Problems of Russian Academy of Sciences:
"Grigory Lazarevich was a remarkable specialist in Representation Theory and a mathematician of the broadest culture, a kind of mathematician who cannot be judged by his publications only. The richness of his inner world, which he would generously share during a discussion in his kitchen or in his room overloaded by books and manuscripts; mathematical ideas and views that you would argue with or agree with; his taste that you would share or use as a starting point; the mere style of his communication, written text, seminar talk — all that now belongs to our memory and is going to be expressed in the work of those who happened to know Grigory Lazarevich closely. Many of us can call ourselves, at least in part, his students."

Marianne Akian and Stéphane Gaubert (Paris, France):
"We first met Grigori in 1994, thanks to Jeremy Gunawardena, at the BRIMS HP-Labs workshop in Bristol on 'Idempotency'. This was the first event in which the Russian and French researchers working on max-plus, idempotent, or tropical structures, could unify their views. Since that time, we saw Grigory regularly in conferences, in particular the ones he was organizing or co-organizing (in Vienna or Moscow). Also, each of us had the chance to visit him. We could appreciate his kindness, his care, the way he was supportive for colleagues, especially the young ones. We understood he was like a mentor in the idempotent analysis community. Of his numerous mathematical contributions, idempotent functional analysis was perhaps the closest to his heart, as it reflected his taste for mathematical foundations. His work in this field had very much influence; or at least, we can say that it influenced the max-plus group of INRIA. We have lost a friend and an insightful mathematician. We will miss him very much."

Vassili Kolokoltsov (Warwick, UK):
"The death of Grigory Litvinov is a great loss to the max-plus community and to all his friends and colleagues. Grisha was a talented mathematician, a very nice person and a friend to be trusted. I will miss him very much."

Ivan Singer (Bucharest, Romania):
"I was fortunate to have contacts with Grigory for many years and to attend some of the conferences organized by him. I will miss him. He was indeed a very nice man and his death is a great loss for our community. May God rest him in peace!"

Peter Butkovič (Birmingham, UK):
"I am deeply saddened by the death of Grigory. He was a great mathematician and a kind person."

Jean-Pierre Quadrat (Tresques, France):
"I will remember all my life the very nice conference in Moscow and all the effort that Grigori has done for us."

Sergeĭ Sergeev (Birmingham, UK):
"Grigori Lazarevich was an intelligent, kind-hearted and unhurried man. However, he had a strong character and he could insist on his point of view. As a mathematician he was deeply understanding, attentive to details and accurate in his proofs.

As a man, he mostly valued honesty, intellect, good humour and lively openness in other people."

Louis Rowen (Bar-Ilan, Israel):
"I only had the privilege of meeting Grigory towards the end of his life, but remember him fondly, together with the great effort he put into making the Moscow seminar of 2012 a success."

María-Jesús de la Puente (Madrid, Spain):
"I met Grigory only once. It was at a conference in Moscow, in June 2011. I found him very kind, lively and good-natured. At that conference I met some important mathematicians, and listened to several very interesting talks on Tropical Mathematics."

Tatjana Grbić (Novi Sad, Serbia):
"Grigory was a nice man and a great mathematician. First of all he was a great friend."

Paola Loreti and Antonio Avantaggiati (Rome, Italy):
"It is with deep great sadness that we learned of Professor Grigory Lazarevich Litvinov's passing. His cleverness, brightness, kindness, and goodness will be always with us."

Gennadi Malaschonok and Colleagues from the University of Tambov (Russia):
"We are deeply saddened about the passing of dear Gregory Lazarevich Litvinov. In recent years, he often gave lectures to students at the Tambov University and has generated some interest in tropical mathematics among them."

Jean Jacques Loiseau (Nantes, France):
"Grigori was indeed a great scientist and an 'honnête homme', as we say in French, both an important intellectual and a real human. The whole community will miss him for years and I'll always remember him."

Nikolaĭ Krivulin (St. Petersburg, Russia):
"I was deeply saddened by the death of Grigory Lazarevich, whom I knew as a very kind and active person, whose works greatly contributed toward the foundation and development of idempotent and tropical mathematics. His scientific heritage is of great importance to us."

Oliver Mason (Maynooth, Ireland):
"While I was not fortunate enough to meet him, I have nothing but admiration for his outstanding contributions to the general area of tropical and idempotent mathematics."

Imran Rashid (Islamabad, Pakistan)
"It was a great honor to me having an email from Grigory Lazarevich Litvinov. On visiting his website I found that he was a great mathematician and a desire arose to start working with him. Now I feel unlucky that I did not get a chance to meet such a great mathematician who is no more among us. No doubt his death is a great loss to our community. May his soul rest in peace."

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This volume contains the proceedings of the International Workshop on Tropical and Idempotent Mathematics, held at the Independent University of Moscow, Russia, from August 26–31, 2012. The main purpose of the conference was to bring together and unite researchers and specialists in various areas of tropical and idempotent mathematics and applications.

This volume contains articles on algebraic foundations of tropical mathematics as well as articles on applications of tropical mathematics in various fields as diverse as economics, electroenergetic networks, chemical reactions, representation theory, and foundations of classical thermodynamics.

This volume is intended for graduate students and researchers interested in tropical and idempotent mathematics or in their applications in other areas of mathematics and in technical sciences.

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