The Influence of Solomon Lefschetz in Geometry and Topology

50 Years of Mathematics at CINVESTAV

Ludmil Katzarkov
Ernesto Lupercio
Francisco J. Turrubiates
Editors
The Influence of Solomon Lefschetz in Geometry and Topology

50 Years of Mathematics at CINVESTAV
The Influence of Solomon Lefschetz in Geometry and Topology
50 Years of Mathematics at CINVESTAV

Ludmil Katzarkov
Ernesto Lupercio
Francisco J. Turrubiartes
Editors

American Mathematical Society
Providence, Rhode Island
Contents

Preface vii

Solomon Lefschetz and Mexico
  Michael Atiyah 1

Recent Progress in Symplectic Flexibility
  Yakov Eliashberg 3

Equivariant Extensions of Differential Forms for Non-compact Lie Groups
  Hugo García-Compeán, Pablo Paniagua, and Bernardo Uribe 19

From Classical Theta Functions to Topological Quantum Field Theory
  Răzvan Gelca and Alejandro Uribe 35

Toric Topology
  Samuel Gitler 69

Beilinson Conjecture for Finite-dimensional Associative Algebras
  Dmitry Kaledin 77

Partial Monoids and Dold-Thom Functors
  Jacob Mostovoy 89

The Weak $b$-principle
  Rustam Sadykov 101

Orbit Configuration Spaces
  Miguel A. Xicoténcatl 113

Dynamical Systems and Categories
  G. Dimitrov, F. Haiden, L. Katzarkov, and M. Kontsevich 133

The Nahm Pole Boundary Condition
  Rafe Mazzeo and Edward Witten 171
Preface

The purpose of this volume is to both celebrate half a century of Mathematics at the Center for Research and Advanced Studies in Mexico City (Cinvestav) and the opening of the Institute for Geometry and Physics Miami-Cinvestav-Campinas.

Cinvestav is one of the most prestigious research centers in Latin America. It focuses most of its activities in exact sciences. It was founded in 1961, and one of the founding departments was the Mathematics Department under the direction of José Adem. At the time, there were two members: Adem, as a full professor, and Samuel Gitler, as an assistant professor (both working in the field of Algebraic Topology). This particular choice was influenced by the fact that both Adem and Gitler were recruited to study their Ph.D.s at Princeton by Solomon Lefschetz, who used to spend half a year in Mexico for many years lecturing at UNAM; he also began a Mexican school of Geometry and Topology. Solomon Lefschetz received the order of the Águila Azteca from President López Mateos in 1964 for his contributions to Mexican Mathematics. Nowadays, the Mathematics Department at CInvestav organizes the Lefschetz Memorial Lecture Series biyearly: the conference is one of the foremost mathematical events in Latin America drawing some of the most distinguished mathematicians in the world to deliver three talks at CInvestav during the fall.

The following is the list of all previous Lefschetz lecturers:

- Jack Hale
- Michael Atiyah
- Enrico Bombieri
- Shing S. Chern
- John Milnor
- David Mumford
- William Thurston
- Ya G. Sinai
- P. Maslov
- Sergio Albeverio
- Maxim Kontsevich
- Dennis Sullivan
- Graeme Segal
- Michael Hopkins
- Edward Witten
- Yasha Eliashberg

This volume contains the paper by Eliashberg corresponding to his 2012 Lefschetz Lectures.
The celebration for the 50th anniversary of the Mathematics Department at Cinvestav took place in 2012 with three main events: TQFT, Langlands and Mirror Symmetry: the Opening Conference of the Institute for Geometry and Physics Miami-Cinvestav-Campinas (IGP-MCC), the 50th year anniversary of Cinvestav, (all together celebrating 50 years of Topology, Geometry and Physics at Cinvestav), and finishing with the International Conference on Symplectic Geometry 2012. This volume contains contributions of the participants from all these events.

The Institute for Geometry and Physics Miami-Cinvestav-Campinas (IGP-MCC) is a joint virtual venture to stimulate a North-South collaboration in Geometry and Physics. The directors are Elisabeth Gasparim, Maxim Kontsevich, Ernesto Lupercio, and Dennis Sullivan, and its founding coincided with the celebrations for the 50th anniversary of Cinvestav.

The events were centered in the fields of Geometry (Algebraic and Symplectic) and Algebraic Topology, which are some of the strongest fields at Cinvestav and, as it turned out to be, they reflect quite nicely that they are still of current interest in the Mathematics Department 50 years after Lefschetz. This volume is thus a witness to his enormous influence in Mexican Mathematics.

Let us describe very briefly the contents of this volume. In the opening paper, Eliashberg surveys the relation between flexibility and rigidity in Symplectic Geometry and Topology, concentrating primarily on the former. This is an important survey of very recent developments in the area by the leading expert.

The second paper by García-Compeán, Paniagua, and Uribe generalizes a result of Witten on the equivalence of absence of anomalies in gauged WZW actions on compact Lie groups to the existence of equivariant extension of the WZW term in the case in which the gauge group is the special linear group.

The third contribution by Gelca and Uribe provides a rigorous construction of Abelian Chern-Simons theory using only the classical theory of theta functions and recovering the Murakami-Ohtsuki-Okada formula for invariants of 3-dimensional manifolds.

The volume also contains a survey paper by Samuel Gitler (a founding member of the Mathematics Department at Cinvestav 50 years ago) dealing with recent developments on the field of Toric Topology by him and his collaborators.

In his paper Kaledin proposes a non-commutative version of Beilinson’s famous conjecture predicting that the regulator map induces an isomorphism between the higher Algebraic $K$-groups of an Arithmetic scheme and its so-called Beilinson’s cohomology. This non-commutative version of the conjecture predicts the existence of a certain exact triangle of complexes associated to smooth and proper dg-Algebras defined over $\mathbb{Z}$, for which each term is related to either (negative or periodic) cyclic homology or Algebraic $K$-theory. The main result of this work is the statement that the non-commutative version of Beilinson’s conjecture holds for finite dimensional Algebras. This is a work in the field of non-commutative Algebraic Geometry.

The paper by Mostovoy generalizes the classical Dold-Thom result in Algebraic Topology from ordinary homology to any generalized homology theory.

Sadykov’s contribution deals with his b-principle. The h-principle is a general observation that differential Geometry problems can often be reduced to problems in (unstable) homotopy theory. Similarly, the b-principle is a general observation that differential Geometry problems can often be reduced to problems in stable
homotopy theory; in his paper, Sadykov surveys some recent developments in the b-principle.

Xicoténcatl contributes to the volume with a survey on orbit configuration spaces. Some of the results deal with the existence of certain natural fibrations extending those of Fadell and Neuwirth, Lie algebras associated to the lower central series of fundamental groups of these spaces, cohomology rings, the Longhoni-Salvatore result of the failure of configuration spaces to preserve homotopy equivalences of closed manifolds, and structures of loop-spaces.

The volume continues with a paper written by Dimitrov, Haiden, Katzarkov, and Kontsevich. This is a paper densely packed with deep results. One of the main objects of study in this contribution is the entropy of automorphisms of non-commutative Algebraic varieties as exemplified by saturated $A_\infty$ categories. The paper is organized around two major results. The first is an explicit identification between the categorical entropy of an endofunctor of the wrapped Fukaya category of a punctured Riemann surface and the classical entropy of a pseudo-Anosov map in the corresponding mapping class. With their second result, the authors give a complete answer to the problem of detecting the density of phases of stable objects in categories of representations of quivers.

Finally, Rafe Mazzeo and Edward Witten conclude this volume with a study of the Nahm pole boundary condition in an ambitious program to generalize the classical Chern-Simons theory of knot invariants.

As the reader can notice, many active fields within Geometry and Topology have been represented as we have celebrated our first 50 years of Mathematics at Cinvestav.

Ernesto Lupercio
Ludmil Katzarkov
Francisco Turrubiates
Selected Published Titles in This Series


614 James W. Cogdell, Freydoon Shahidi, and David Soudry, Editors, Automorphic Forms and Related Geometry, 2014

612 Patricio Cifuentes, José García-Cuerva, Gustavo Garrigós, Eugenio Hernández, José María Martell, Javier Parcet, Keith M. Rogers, Alberto Ruiz, Fernando Soria, and Ana Vargas, Editors, Harmonic Analysis and Partial Differential Equations, 2014

611 Robert Fitzgerald Morse, Daniela Nikolova-Popova, and Sarah Witherspoon, Editors, Group Theory, Combinatorics, and Computing, 2014

610 Pavel Etingof, Mikhail Khovanov, and Alistair Savage, Editors, Perspectives in Representation Theory, 2014


607 Kiyoshi Igusa, Alex Martsinkovsky, and Gordana Todorov, Editors, Expository Lectures on Representation Theory, 2014

606 Chantal David, Matilde Lalín, and Michelle Manes, Editors, Women in Numbers 2, 2013

605 Omid Amini, Matthew Baker, and Xander Faber, Editors, Tropical and Non-Archimedean Geometry, 2013

604 José Luis Monta˜na and Luis M. Pardo, Editors, Recent Advances in Real Complexity and Computation, 2013


602 Vyjayanthi Chari, Jacob Greenstein, Kailash C. Misra, K. N. Raghavan, and Sankaran Viswanath, Editors, Recent Developments in Algebraic and Combinatorial Aspects of Representation Theory, 2013


598 Eric Todd Quinto, Fulton Gonzalez, and Jens Gerlach Christensen, Editors, Geometric Analysis and Integral Geometry, 2013


596 Khodr Shamseddine, Editor, Advances in Ultrametric Analysis, 2013


For a complete list of titles in this series, visit the AMS Bookstore at www.ams.org/bookstore/conmseries/.
The influence of Solomon Lefschetz (1884–1972) in geometry and topology 40 years after his death has been very profound. Lefschetz’s influence in Mexican mathematics has been even greater. In this volume, celebrating 50 years of mathematics at Cinvestav-México, many of the fields of geometry and topology are represented by some of the leaders of their respective fields.

This volume opens with Michael Atiyah reminiscing about his encounters with Lefschetz and México. Topics covered in this volume include symplectic flexibility, Chern–Simons theory and the theory of classical theta functions, toric topology, the Beilinson conjecture for finite-dimensional associative algebras, partial monoids and Dold–Thom functors, the weak b-principle, orbit configuration spaces, equivariant extensions of differential forms for noncompact Lie groups, dynamical systems and categories, and the Nahm pole boundary condition.