Recent Advances in Representation Theory, Quantum Groups, Algebraic Geometry, and Related Topics
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AMS Special Sessions on Geometric and Algebraic Aspects of Representation Theory and Quantum Groups and Noncommutative Algebraic Geometry
October 13–14, 2012
Tulane University, New Orleans, LA

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Editors
# Contents

Preface vii  

Special session on “Geometric and algebraic aspects of representation theory” organized by Pramod Achar and Dijana Jakelić ix  

Special session on “Quantum groups and noncommutative algebraic geometry” organized by Kailash C. Misra and Milen Yakimov xi  

A classification of irreducible Wakimoto modules for the affine Lie algebra $A_{1}^{(1)}$  
DRAŽEN ADAMOVIĆ 1  

A note on $U_q(D_4^{(3)})$-Demazure crystals  
ALYSSA M. ARMSTRONG and KAILASH C. MISRA 13  

Dimensions of imaginary root spaces of hyperbolic Kac–Moody algebras  
LISA CARBONE, WALTER FREYN, and KYU-HWAN LEE 23  

On some structures of Leibniz algebras  
ISMAIL DEMIR, KAILASH C. MISRA, and ERNIE STITZINGER 41  

A geometric construction of generalized $q$-Schur algebras  
STEPHEN DOTY and YIQIANG LI 55  

On the classification of irreducible Gelfand–Tsetlin modules of $sl(3)$  
VYACHESLAV FUTORNY, DIMITAR GRANTCHAROV, and LUIS E. RAMIREZ 63  

Supersymmetry and the modular double  
IVAN CHI-HO IP and ANTON M. ZEITLIN 81  

On Weyl modules for quantum and hyper loop algebras  
DIJANA JAKELIĆ and ADRIANO MOURA 99  

Toroidal Lie superalgebras and free field representations  
NAIHUAN JING and CHONGBIN XU 135  

Invariants of $(-1)$-skew polynomial rings under permutation representations  
ELLEN KIRKMAN, JAMES KUZMANOVICH, and JAMES J. ZHANG 155  

On total Frobenius-Schur indicators  
GONGXIANG LIU and SIU-HUNG NG 193  

Loop Grassmannians in the framework of local spaces over a curve  
IVAN MIRKOVIĆ 215
Decorated geometric crystals and polyhedral realization of type $D_n$
    Toshiki Nakashima 227

Some Koszul properties of standard and irreducible modules
    Brian J. Parshall and Leonard L. Scott 243

On higher order Leibniz identities in TCFT
    Anton M. Zeitlin 267
Preface

Representation theory, along with its interactions with other areas of mathematics, such as noncommutative algebra, algebraic geometry, and mathematical physics, constitutes a major area of current mathematical research. Many aspects of the traditional theory of finite-dimensional semisimple Lie algebras, due to Killing, Cartan, Weyl, and others around the turn of the twentieth century, have found vast generalizations in recent decades in a number of different directions such as Kac–Moody algebras (especially affine Lie algebras), vertex algebras, and “quantum” and “super” versions of these algebras. In the case of quantum algebras in particular, classical Lie theory makes contact both with the abstract study of Hopf algebras and general techniques from noncommutative ring theory. Quantum analogues of varieties such as affine spaces, tori, and Schubert cells play a key role here. This brings us to connections with ordinary (commutative) geometry. A number of major advances in representation theory in recent decades have been achieved by translating algebraic questions into terms involving either algebraic topology (via perverse sheaves or \(D\)-modules) or algebraic geometry (via coherent sheaves and \(K\)-theory) of flag varieties, quiver varieties, and related spaces.

At the Fall Southeastern Section Meeting of the American Mathematical Society in New Orleans, LA, during October 13–14, 2012, two special sessions on: “Geometric and Algebraic Aspects of Representation Theory” and “Quantum Groups and Noncommutative Algebraic Geometry,” respectively, were organized by the editors. These were mathematically robust and engaging sessions in which a number of senior and junior researchers presented their work on a variety of research topics. Several of the speakers of these two special sessions have contributed to this volume. We believe that the developments brought together in this volume constitute a valuable contribution to the literature in representation theory. The diversity of topics covered in this volume will help readers working in one branch of representation theory to discover connections to other branches of which they may not have otherwise been aware. This volume is a useful reference to both established mathematicians and junior researchers in representation theory.

We would like to thank all the speakers at these two special sessions, especially those who have contributed to this proceedings volume. We are grateful to all the referees for their thoughtful and timely reviews. We would also like to thank Edward Dunne for suggesting that we put this proceedings together, and Christine Thivierge for her guidance and help through the editorial process.

The Editors
Special session on “Geometric and Algebraic Aspects of Representation Theory” organized by Pramod Achar and Dijana Jakelić

Dražen Adamović (University of Zagreb, Croatia), *On representations of affine vertex algebras outside the category* $\mathcal{O}$.

Bojko Bakalov (North Carolina State University), *W*-constraints for the total descendant potential of a simple singularity.

Martina Balagović (University of York, UK), *Category* $\mathcal{O}$ *for rational Cherednik algebras in positive characteristic.*

Roman Bezrukavnikov (Massachusetts Institute of Technology), *Character sheaves on loop groups and endoscopy.*

Brian Boe (University of Georgia), *Thick subcategories for classical Lie superalgebras.*

Merrick Brown (The University of North Carolina at Chapel Hill), *Saturation in tensor product decomposition of integrable affine* $\mathfrak{sl}_2$ *representations.*

Sean Clark (University of Virginia), *Towards canonical bases for quantum* $\mathfrak{kac–moody}$ *superalgebras.*

Ben Cox (College of Charleston), *Virasoro action on Imaginary Verma modules and the operator form of the KZ-equation.*

Houssein El Turkey (University of Oklahoma), *Presenting Schur superalgebras.*

Jörg Feldvoss (University of South Alabama), *Split abelian chief factors and Lie algebra cohomology.*

Dimitar Grantcharov (University of Texas at Arlington), *Weight modules of infinite dimensional Weyl algebras.*

Naihuan Jing (North Carolina State University), *Vertex operators and Jack polynomials.*

Jonathan Kujawa (University of Oklahoma), *The Generalized Kac–Wakimoto conjecture.*

Shrawan Kumar (The University of North Carolina at Chapel Hill), *Positivity in* $T$-*equivariant* $K$-*theory of flag varieties associated to* $\mathfrak{kac–moody}$ *groups.*

Yiqiang Li (University at Buffalo—SUNY), *A geometric realization of modified quantum algebras.*

Myron Minn-Thu-Aye (Louisiana State University), *Multiplicity formulas for perverse coherent sheaves on the nilpotent cone.*

Ivan Mirković (University of Massachusetts, Amherst), *Geometry of modular representation theory and Koszul duality.*
Kailash C. Misra (North Carolina State University), $A_\alpha^{(1)}$-geometric crystal corresponding to Dynkin index $i = 2$ and its ultradiscretization.

Adriano Moura (University of Campinas, Brazil), Prime representations and self extensions of representations of quantum affine algebras.

Toshiki Nakashima (Sophia University, Japan), Decorated geometric crystals and monomial realizations of crystals.

Brian Parshall (University of Virginia), Shifted generic cohomology.

Leonard Scott (University of Virginia), Forced gradings and p-filtrations.

Anton Zeitlin (Columbia University), On the continuous series for $\hat{sl}(2,\mathbb{R})$. 
Special session on “Quantum Groups and Noncommutative Algebraic Geometry” organized by Kailash C. Misra and Milen Yakimov

Ben Cox (College of Charleston), \textit{DJKM algebras, their universal central extension and orthogonal polynomials.}

Naihuan Jing (North Carolina State University), \textit{Principal realizations of the Yangian Y(sl(n)).}

Garrett Johnson (The Catholic University of America), \textit{Quantum affine Schubert cells and FRT-bialgebras.}

Rajesh Kulkarni (Michigan State University), \textit{Representations of Clifford algebras and Ulrich bundles on hypersurfaces.}

Aaron Lauda (University of Southern California), \textit{Odd structures arising from categorified quantum groups.}

Kyu-Hwan Lee (University of Connecticut), \textit{Eisenstein series on affine Kac–Moody groups over function fields.}

Yiqiang Li (University at Buffalo—SUNY), \textit{On quantum matrix algebras.}

Martin Lorenz (Temple University), \textit{Torus actions on noncommutative algebras.}

Susan Montgomery (University of Southern California), \textit{Computing Frobenius-Schur indicators for doubles of groups.}

Siu-Hung Ng (Iowa State University), \textit{Total indicators of the representations of quasi-Hopf algebras.}

Anne Schilling (University of California, Davis), \textit{A uniform combinatorial model for Kirillov-Reshetikhin crystals and specialized Macdonald polynomials.}

Chelsea Walton (Massachusetts Institute of Technology), \textit{Quantum binary polyhedral groups and their actions on quantum planes.}

Harold Williams (University of California, Berkeley), \textit{Cluster ensembles and the Chamber Ansatz.}

Anton Zeitlin (Columbia University), \textit{Algebraic structures of stringy sigma models and homotopy algebras.}

James Zhang (University of Washington), \textit{Invariant theory of finite group actions on down-up algebras.}
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This volume contains the proceedings of two AMS Special Sessions “Geometric and Algebraic Aspects of Representation Theory” and “Quantum Groups and Noncommutative Algebraic Geometry” held October 13–14, 2012, at Tulane University, New Orleans, Louisiana.

Included in this volume are original research and some survey articles on various aspects of representations of algebras including Kac–Moody algebras, Lie superalgebras, quantum groups, toroidal algebras, Leibniz algebras and their connections with other areas of mathematics and mathematical physics.