CONTEMPORARY MATHEMATICS

626

Operator Methods in Wavelets, Tilings, and Frames

AMS Special Session
Harmonic Analysis of Frames, Wavelets, and Tilings
April 13–14, 2013
Boulder, Colorado

Veronika Furst
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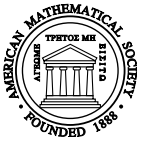

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Contents

Preface

Frames were first introduced by Duffin and Schaeffer in 1952 in the context of nonharmonic Fourier series but have enjoyed widespread interest in recent years, particularly as a unifying concept. Indeed, mathematicians with backgrounds as diverse as classical and modern harmonic analysis, Banach space theory, operator algebras, and complex analysis have recently worked in frame theory. The present volume contains papers expositing the theme of operator theoretic methods in frame theory in four specific contexts: frame constructions, wavelet theory, tilings, and sampling theory.

There are numerous constructions of frames, as there are numerous situations in which frame theory plays a central role, and each of these situations requires a frame with different characteristics. The paper by Casazza and Woodland discusses frame constructions, and associated projections, which allow for the reconstruction of an unknown vector using the magnitude of frame coefficients without the phase. The paper by Kutyniok, Okoudjou, and Phillips concerns frames which can be preconditioned via scalar multiplication to obtain a tight frame. The paper by Han, Larson, Liu and Liu approaches the idea of a frame in a generalized sense, in which the frame is given by a set of operators, not a set of vectors.

Although the first wavelet was introduced by Haar in 1909, wavelet analysis officially took off with the pioneering work of Daubechies, Grossman, and Meyer in the 1980s. The main attractiveness of a wavelet is its simultaneous localization of a square-integrable function in both time and frequency. Its "zooming" capability is formalized in the definition of a multiresolution analysis. Ghandehari and Taylor generalize the classical dilation and translation operators by considering a unitary representation of a locally compact group G and defining a wavelet to be a vector in the associated Hilbert space for which a reconstruction formula holds in a weak sense. Their focus is how the images of the corresponding continuous wavelet transform, as subspaces of $L^2(G)$, change and are related to one another, as a consequence of varying the wavelet. The paper by Currey, Mayeli, and Oussa also generalizes the wavelet representation of the subgroup of the $ax + b$ group that is isomorphic to the subgroup of unitary operators generated by the classical dilation and translation. The authors replace the Hilbert space $L^2(\mathbb{R}^n)$ by $L^2(N)$ for a simply connected, connected nilpotent Lie group N. They define a corresponding wavelet representation and analyze its direct integral decomposition, particularly for non-commutative N. In the paper by Massopust, exponential splines of complex order extend the class of exponential B-splines of order n for $n \in \mathbb{N}$ and polynomial B-splines of complex order. The new class of splines defines multiresolution analyses of $L^2(\mathbb{R})$ and corresponding wavelet bases.

The Fuglede conjecture from 1974 presents the connection that is often, but not always, present between sets that tile $\mathbb{R}^d$ by translation and the existence of spectral sets associated with the tiling. The conjecture is resolved for at least dimension 3 but not dimensions 2 and 1. The paper by Dutkay and Hausserman considers tiling sets in dimension 1. The authors present properties of unitary groups of local translations acting on subsets of the real line and draw connections to tilings. The Fuglede conjecture created increased interest in the presence or lack of Fourier bases or Fourier frames with respect to a variety of measures. The paper by Jorgensen, Kornelson, and Shuman presents spectra on a fractal measure space and gives structural information about the connections between different spectra on the same space.

Sampling theory concerns the reconstruction of an unknown function from its known samples at certain points in its domain. This idea can be traced back to Cauchy, where the unknown function was a trigonometric polynomial, but in the modern context, sampling theory can be described in terms of frames. In this form, the main problem is when a certain operator possesses a generalized inverse. The paper by al-Sa'di and Weber gives necessary conditions which guarantee that this operator does possess a generalized inverse, where the unknown function belongs to a Hilbert space of entire functions. The papers by Aceska and Tang, and by Davis concern the variation on sampling theory in which some of the known samples of the unknown function are obtained after an operator acts upon the function. In the paper by Aceska and Tang, the space of functions is a hybrid shift invariant space, and the operator which acts in between successive sampling operations is a convolution operator. In the paper by Davis, the function space of square-summable sequences, and the operator acting between sampling operations may involve a nonlinear forcing term. In both papers, the essential question is: When does the matrix representation for an operator possess an appropriate submatrix with a generalized inverse?

This collection of papers covers a wide variety of topics, including: convex geometry, direct integral decompositions, Beurling density, operator-valued measures, splines, and more. These topics arise naturally in the study of frames, which again is the unifying theme in this volume. In nearly all of the papers, ideas and results from operator theory are the crucial tools in solving the problems in the study of frames. This volume will be of interest to researchers in frame theory, and also to those in approximation theory, representation theory, functional analysis, and harmonic analysis.

Veronika Furst
Keri Kornelson
Eric Weber

Participants

Speakers and titles from the AMS Special Session "Harmonic Analysis of Frames, Wavelets, and Tilings" from the AMS Western Sectional Meeting, Boulder, CO, April 13–14, 2013.

Marcin Bownik
Existence of Frames with Prescribed Norms and Frame Operator

Peter G. Casazza
Fusion Frames for Wireless Sensor Networks

Jacqueline Davis
Dynamical Sampling

Dorin Dutkay
The Fuglede Conjecture in Dimension One

Matthew Fickus
Characterizing Completions of Finite Frames

Deguang Han
Spectrally Optimal Frames for Erasures

John Haussermann
Tiling Properties of Spectra of Measures

John Jasper
Spectra of Frame Operators with Prescribed Frame Norms

Palle Jorgensen
Tilings in Wavelet Theory: IFS Measures and Wavelet Packets

Chun-Kit Lai
Spectral Property of Cantor Measures with Consecutive Digits

David R. Larson
Frames, Dilations and Operator-Valued Measures

Peter Massopust
Exponential Splines with Complex Order

Azita Mayeli
Bracket Map for the Heisenberg Group and the Characterization of Cyclic Subspaces

Kathy D. Merrill
Simple n-Dimensional Wavelet Sets

Vignon S Oussa
Parseval Frame Wavelets on Some Non-Abelian Nilpotent Matrix Groups

Gabriel Picioroaga
Orthonormal Bases Generated by Cuntz Algebras

Benjamin Purkis
Constructing Projective Multiresolution Analyses over Irrational Rotation Algebras

Darrin Speegle
Linear Independence of Time-Frequency Translates of Functions with Faster than Exponential Decay

Tim Wertz
Localization of Matrix Factorizations

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626 **Veronika Furst, Keri A. Kornelson, and Eric S. Weber, Editors,** Operator Methods in Wavelets, Tilings, and Frames, 2014

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Wavelets, Tilings, and Frames • Furst et al., Editors

This volume contains the proceedings of the AMS Special Session on Harmonic Analysis of Frames, Wavelets, and Tilings, held April 13–14, 2013, in Boulder, Colorado.

Frames were first introduced by Duffin and Schaeffer in 1952 in the context of nonharmonic Fourier series but have enjoyed widespread interest in recent years, particularly as a unifying concept. Indeed, mathematicians with backgrounds as diverse as classical and modern harmonic analysis, Banach space theory, operator algebras, and complex analysis have recently worked in frame theory. Frame theory appears in the context of wavelets, spectra and tilings, sampling theory, and more.

The papers in this volume touch on a wide variety of topics, including: convex geometry, direct integral decompositions, Beurling density, operator-valued measures, and splines. These varied topics arise naturally in the study of frames in finite and infinite dimensions. In nearly all of the papers, techniques from operator theory serve as crucial tools to solving problems in frame theory.

This volume will be of interest not only to researchers in frame theory but also to those in approximation theory, representation theory, functional analysis, and harmonic analysis.

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