CONTEMPORARY MATHEMATICS

630

Centre de Recherches Mathématiques Proceedings

Geometric and Spectral Analysis

CRM Workshops on:
Geometry of Eigenvalues and Eigenfunctions
June 4–8, 2012

Manifolds of Metrics and Probabilistic Methods
in Geometry and Analysis
July 2–6, 2012

Spectral Invariants on Non-compact and Singular Spaces
July 23–27, 2012
Centre de Recherches Mathématiques,
Université de Montréal, Quebec, Canada

Pierre Albin
Dmitry Jakobson
Frédéric Rochon
Editors

American Mathematical Society
Providence, Rhode Island

Centre de Recherches Mathématiques
Montréal, Québec, Canada

Geometric and Spectral Analysis

CONTEMPORARY MATHEMATICS

630

Centre de Recherches Mathématiques Proceedings

Geometric and Spectral Analysis

CRM Workshops on:
Geometry of Eigenvalues and Eigenfunctions
June 4–8, 2012

Manifolds of Metrics and Probabilistic Methods
in Geometry and Analysis
July 2–6, 2012

Spectral Invariants on Non-compact and Singular Spaces
July 23–27, 2012
Centre de Recherches Mathématiques,
Université de Montréal, Quebec, Canada

Pierre Albin
Dmitry Jakobson
Frédéric Rochon
Editors

American Mathematical Society
Providence, Rhode Island

Centre de Recherches Mathématiques
Montréal, Québec, Canada

2010 *Mathematics Subject Classification.* Primary 58J05, 58J40, 58J50, 58J52, 53C55, 60D05, 49R05, 46E35, 53A30, 35Q41.

Library of Congress Cataloging-in-Publication Data

Geometric and spectral analysis / Pierre Albin, Dmitry Jakobson, Frédéric Rochon, editors.
p. cm. – (Contemporary mathematics ; volume 630)
Centre de Recherches Mathématiques Proceedings.
CRM Workshops on: Geometry of Eigenvalues and Eigenfunctions, June 4–8, 2012, Manifolds of Metrics and Probabilistic Methods in Geometry and Analysis, July 2–6, 2012, Spectral Invariants on Non-compact and Singular Spaces, July 23–27, 2012, Centre de Recherches Mathématiques, Université de Montreal, Montréal, Quebec, Canada.
Includes bibliographical references.
ISBN 978-1-4704-1043-8 (alk. paper)
1. Spectrum analysis–Congresses. 2. Geometric analysis–Congresses. I. Albin, Pierre, 1976– II. Jakobson, Dmitry, 1970– III. Rochon, Frédéric, 1978–.
QC20.7.S64G46 2014
516–dc23 2014021478

Contemporary Mathematics ISSN: 0271-4132 (print); ISSN: 1098-3627 (online)

DOI: http://dx.doi.org/10.1090/conm/630

Contents

Preface

In 2012, thematic programs on *Geometric Analysis and Spectral Theory* and on *Moduli spaces, Extremality and Global Invariants* were held at the CRM. There were several workshops on cutting-edge topics in geometric analysis and spectral theory, notably on the *Geometry of Eigenvalues and Eigenfunctions*, on *Manifolds of Metrics and Probabilistic Methods in Geometry and Analysis* and on *Spectral invariants on non-compact and singular spaces*. This was the occasion to learn about the most recent developments in various interconnected fields and also the starting point for new investigations and fruitful collaborations.

The 2012 Spring Semester on Geometric Analysis and Spectral Theory at the CRM brought together researchers working in these fields interpreted in a broad sense, so as to include applications to fundamental problems in geometry, PDE, dynamical systems and mathematical physics. The present CRM Proceedings includes articles written by participants of the workshops in *Geometry of Eigenvalues and Eigenfunctions* and on *Manifolds of Metrics and Probabilistic Methods in Geometry and Analysis*.

The workshop on Geometry of Eigenvalues and Eigenfunctions was held at the CRM on June 4–8, 2012. It was organized by D. Jakobson (McGill) and I. Polterovich (Montréal). The workshop brought together the leading researchers and young mathematicians working in various areas of geometric spectral theory. Many problems in the field are motivated by questions originating in the study of real-life phenomena: quantum-mechanical effects, vibration of membranes and plates, oscillations of fluids, etc. One of the main highlights of the meeting was a series of Aisenstadt lectures on Quantum Unique Ergodicity by Elon Lindenstrauss. Quantum Ergodicity and properties of eigenfunctions in the semiclassical limit were among the central themes of the workshop. In recent work, Jakobson, Safarov and Strohmaier analyzed the semiclassical limit of spectral theory on manifolds whose metrics have jump-like discontinuities; the corresponding semiclassical limit does not relate to a classical flow but rather to branching (ray-splitting) billiard dynamics. A Quantum Ergodicity theorem was established for discontinous systems, and a new notion of ergodicity for the ray-splitting dynamics was introduced, and a class of examples of such ergodic systems was provided by Yves Colin de Verdière. Very important technical results in that work were established in the paper by Y. Safarov appearing in these proceedings. His paper discusses Fourier integral operators (FIOs) associated with canonical transformations. It is well known that the composition of two such operators is a FIO associated with the composition of the corresponding transformations. It is also known that the principal symbol of a FIO is a half-density with values in a linear bundle over a Lagrangian manifold. The latter makes it impossible to write down a simple formula for the principal symbol

of the composition: such a formula must involve the product of principal symbols but it is not a half-density. It will be shown that, under a slightly different definition of the principal symbol, this problem can be avoided, and then the formulae for principal symbols of compositions and adjoint operators become almost as simple as for pseudodifferential operators.

Another topic considered at the conference was the behavior of eigenfunctions of the Laplacian on Riemannian manifolds and Euclidean domains. In their paper appearing in these proceedings, B. Nguyen, A. Delytsin, and D. Grebenkov discuss the eigenvalue problem for the Laplace operator in a planar domain which can be decomposed into a bounded domain of arbitrary shape and elongated "branches" of variable cross-sectional profiles. They show that when the eigenvalue is smaller than a prescribed threshold, the corresponding eigenfunction decays exponentially along each branch.

Two papers appearing in these proceedings concern the behavior of eigenvalues of differential operators. In the paper by V. Guillemin, E. Legendre, and R. Sena-Dias, the authors calculate the variation of the Rayleigh quotient under conformal variations and under variations within a Kähler class. They use the conformal formula to reprove Uhlenbeck's result: a generic metric can be perturbed by a conformal factor into a metric whose eigenspaces for the Laplace operator have dimension 1. In the case of a Kähler manifold, they write an explicit variational formula for the Rayleigh quotient under Kähler deformations and discuss this relationship with the following question: Is the spectrum of a generic Kähler metric simple?

G. Poliquin's paper considers lower bounds for the principal frequency of the p-Laplacian on n-dimensional Euclidean domains. In particular, he extends the classical results involving the inner radius of a domain and the first eigenvalue of the Laplace operator to the case $p \neq 2$. As a by-product, he obtains a lower bound on the size of the nodal set of an eigenfunction of the p-Laplacian on planar domains.

The workshop on Manifolds of Metrics and Probabilistic Methods in Geometry and Analysis was held at CRM on July 2–6, 2012. It was organized by D. Jakobson (McGill), S. Klevtsov (ULB), and S. Zelditch (Northwestern). The workshop brought together mathematicians and physicists working on questions related to random geometry in a broad sense.

The study of geometry of the infinite-dimensional space of Kähler metrics was the subject of many talks at the conference, as well as two papers in these proceedings. The main result in the paper of R. Berman provides a new formula for the distribution of the exponentially small eigenvalues of Dolbeault Laplacians associated to high tensor powers of a line bundle over a compact complex manifold, which in physics terminology is a measure of "tunneling" of the Dolbeault complex. Along the way a new proof of the asymptotics of the induced Quillen metric on the corresponding determinant line is obtained. A brief comparison with the tunneling effect for Witten Laplacians and large deviation principles for fermions is also made.

A long paper by Y. Rubinstein provides a survey on Kähler-Einstein metrics in the smooth and singular (edge) setting, presented from a unifed point of view. The paper takes the reader from the very beginning of Kähler geometry and the Einstein equations, through the edge calculus for Kähler-Einstein edge metrics as developed in the work of Rubinstein, Jeffres and Mazzeo, over the geometry of the space of Kähler metrics and its associated functionals, to the very recent work of

Rubinstein-Cheltsov on asymptotic log Fano varieties. The survey is concluded with a conjectural picture concerning a generalization of the Calabi conjecture in the asympotic log setting.

Several sessions at the workshop were devoted to the lectures on Liouville 2d quantum gravity and the physical definition of random metrics. L. Chen outlined a construction (jointly with D. Jakobson) of random measures in dimension four, inspired by the construction of Duplantier and Sheffield in dimension two, and described an approach to derive a KPZ-type relation for spherical averages of those measures. The construction used a conformally covariant differential operator (Paneitz operator) in dimension four, in place of the Laplacian used in dimension two. R. Ponge gave an overview of Fefferman's program, conformally covariant differential operators (including the Yamabe and the Paneitz operators). In his paper appearing in these proceedings, Ponge studied the logarithmic singularities of the Green functions of conformally covariant operators, including the Yamabe and Paneitz operators, as well as the conformal fractional powers of the Laplacian arising from scattering theory for Poincaré-Einstein metrics. He explains how to compute explicitly the logarithmic singularities of the Green functions of the conformal powers of the Laplacian. The results are formulated in terms of Weyl conformal invariants arising from the ambient metric of Fefferman-Graham. As applications we obtain characterizations in terms of Green functions of locally conformally flat manifolds and a spectral theoretic characterization of the conformal class of the round sphere.

Several talks at the conference concerned the study of random functions. L. Nicolaescu's talk described his results on critical values of random functions on a given compact Riemannian manifold. In his paper appearing in these proceedings, Nicolaescu surveys his work on the distribution of critical points and critical values of those functions, defined as a Fourier-like eigenseries whose coefficients are independent Gaussian random variables.

Nicolas Burq gave a talk on probabilistic Sobolev embeddings, showing that, from a PDE point of view, randomly chosen functions may behave much more nicely than what the deterministic theory would predict. In the paper by N. Burq and G. Lebeau appearing in these proceedings, the authors present some probabilistic versions of Sobolev embeddings and an application to the growth rate of the L^p norms of spherical harmonics, as well as eigenfunctions on manifolds with transitive isometry group actions.

Shortly after these workshops, the CRM thematic year on *Moduli spaces, extremality and global invariant* started with its first workshop on *Spectral invariants on non-compact and singular spaces*. It was held on July 23–27 at the CRM and was organized by P. Albin (UIUC) and F. Rochon (UQAM). Since it involved spectral theory and geometric analysis, as well as global invariants and moduli spaces (which are a good source of examples of singular spaces), this workshop was a natural transition between the two thematic programs. A good part of it was concerned with spectral invariants arising in index theory, like the eta invariant, analytic torsion or the determinant of Laplacian. The latter also arises naturally in scattering theory and more generally in spectral theory, which was the other important theme of the workshop.

The workshop was the starting point of an interesting collaboration between Dean Baskin, Jeremy Marzuola, and Jared Wunsch, who obtained global Strichartz

estimates for exterior polygonal domains as well as some smoothing estimates. These results have natural applications to the study of some non-linear Schrödinger equations on exterior polygonal domains. Their results appear in the present proceedings as one of the five articles related to this workshop.

There is also the collaboration of Zhiqin Lu and Julie Rowlett, which provides a careful analysis of the gap between the first two positive eigenvalues of the Laplacian of a convex domain in $\mathbb{R}^n$ that degenerates to a domain in $\mathbb{R}^{n-1}$. One of the important conclusions is that this gap detects the geometry of a one-dimensional collapse. In fact, depending on the geometry of the collapse, the gap can diverge or remain bounded.

In another direction, Maxim Braverman and Boris Vertman give a new proof of the Bismut-Zhang formula for the ratio of the analytic torsion and the Reidemeister torsion. Their proof works when the representation of the fundamental group lies in a connected component of a unitary representation and has the advantage of being significantly simpler. The idea is to use the Cheeger-Müller theorem for the unitary representations and extend the result to the whole connected component using analytic continuation.

Gerardo A. Mendoza contributes to these proceedings with a nice paper on b-complex manifolds, which are complex manifolds arising from a complex structure of the b-tangent bundle of Melrose. This is a subject he has developed over the years. In the paper published in these proceedings, Mendoza studies the rich structure of the cohomology of the indicial complex obtained by restricting the Dolbeault complex to the boundary.

Finally, B.-W. Schulze gives a valuable and useful overview of the iterative approach, an approach he and his collaborators developed to construct and study a pseudodifferential calculus suitable for doing analysis on stratified spaces.

Pierre Albin
Dmitry Jakobson
Frédéric Rochon

CONM 630

The Centre de Recherches Mathématiques (CRM) was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM receives funding from the Natural Sciences and Engineering Research Council (Canada), the FRQNT (Québec), the NSF (USA), and its partner universities (Université de Montréal, McGill, UQAM, Concordia, Université Laval, Université de Sherbrooke and University of Ottawa).

Geometric and Spectral Analysis • Albin et al., Editors

In 2012, the Centre de Recherches Mathématiques was at the center of many interesting developments in geometric and spectral analysis, with a thematic program on Geometric Analysis and Spectral Theory followed by a thematic year on Moduli Spaces, Extremality and Global Invariants.

This volume contains original contributions as well as useful survey articles of recent developments by participants from three of the workshops organized during these programs: Geometry of Eigenvalues and Eigenfunctions, held from June 4–8, 2012; Manifolds of Metrics and Probabilistic Methods in Geometry and Analysis, held from July 2–6, 2012; and Spectral Invariants on Non-compact and Singular Spaces, held from July 23–27, 2012.

The topics covered in this volume include Fourier integral operators, eigenfunctions, probability and analysis on singular spaces, complex geometry, Kähler-Einstein metrics, analytic torsion, and Strichartz estimates.

American Mathematical Society
www.ams.org

Centre de Recherches Mathématiques
www.crm.math.ca

ISBN 978-1-4704-1043-8

9 781470 410438

CONM/630

AMS/CRM