

# CONTEMPORARY MATHEMATICS

668

## Probability on Algebraic and Geometric Structures

International Research Conference  
in Honor of Philip Feinsilver, Salah-Eldin A. Mohammed,  
and Arunava Mukherjea  
Probability on Algebraic and Geometric Structures  
June 5–7, 2014  
Southern Illinois University, Carbondale, Illinois

Gregory Budzban  
Harry Randolph Hughes  
Henri Schurz  
Editors



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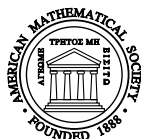
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An international research conference celebrating the distinguished careers of  
Philip Feinsilver, Salah Mohammed, and Arunava Mukherjea.



## Contents

Preface	ix
List of participants	xi
The fine structure of the stationary distribution for a simple Markov process ANDREAS ANCKAR and GÖRAN HÖGNÄS	1
Superdegenerate hypoelliptic differential operators DENIS BELL	13
Some remarks on the convolution equation $\mu * \beta = \mu$ and product semigroups GREG BUDZBAN and ARUNAVA MUKHERJEA	21
Limit distributions of products of I.I.D. random $2 \times 2$ stochastic matrices: an open problem SANTANU CHAKRABORTY	31
Calculus via regularizations in Banach spaces and Kolmogorov-type path-dependent equations ANDREA COSSO, CRISTINA DI GIROLAMI, and FRANCESCO RUSSO	43
Matrices with zero row sums, tree theorems and a Markov chain on trees PHILIP FEINSILVER	67
Convolution of orbital measures on symmetric spaces: A survey P. GRACZYK and P. SAWYER	81
Exit times for some autoregressive processes with non-Gaussian noise distributions GÖRAN HÖGNÄS and BRITA JUNG	111
Brief history of optimal control theory and some recent developments ISABELLE KEMAJOU-BROWN	119
Krawtchouk matrices, Feynman path integral and the split quaternions JERZY KOCIK	131
The Itô-Stratonovich formula for an operator of order four RÉMI LÉANDRE	165
Random walks on motion groups C. R. E. RAJA and R. SCHOTT	171



Existence, uniqueness, and energy of modified stochastic sine-Gordon equation with multiplicative noise on one-dimensional domain HENRI SCHURZ and ABDALLAH M. TALAFHA	179
Squared Bessel process with delay LOCHANA SIRIWARDENA and HARRY RANDOLPH HUGHES	199
Kravchuk matrices and induced operators on Clifford algebras G. STACEY STAPLES	207

## Preface

On June 5–7th, 2014, researchers from the United States, Canada, and Europe gathered in Carbondale, Illinois to celebrate the distinguished careers of three mathematicians who had each recently retired. All three have made important and deep contributions to the chosen areas of research.

Philip Feinsilver received his Ph.D. in 1975 from NYU, Courant where his advisor was S. R. S Varadhan. His wide-ranging contributions include results in probability on algebraic structures, stochastic processes, and representation theory. In addition, he made significant contributions to the understanding of Krawtchouk polynomials and matrices.

Salah-Eldin Mohammed received his Ph.D. in 1976 from the University of Warwick where his advisor was James Eells. His research contributions include important results in deterministic functional differential equations on manifolds, stochastic flows, and the Malliavin calculus. His book on stochastic functional differential equations included groundbreaking work on stochastic systems with memory. His work on the the Stable Manifold Theorem with Zhang and Zhou appeared in the *Memoirs of the American Mathematical Society*.

Arunava Mukherjea received his Ph.D. in 1967 from Wayne State University where his advisor was A. T. Barucha-Reid. He is the foremost authority in the area of probability on topological semigroups. He has made important contributions to the convergence behavior of non-homogeneous Markov chains, the structure of idempotent matrices, and analysis of multi-variate normal distributions. In addition, he is the founding editor of the *Journal of Theoretical Probability*.

The editors would like to thank the conference organizing committee for their work: Jerzy Kocik, Southern Illinois University Carbondale; John McSorley, Southern Illinois University Carbondale; and G. Stacey Staples, Southern Illinois University Edwardsville. In addition, the editors want to express their appreciation to Mr. Charles Gibson for his work in organizing and formatting the submissions to the volume.

Gregory Budzban  
Harry Randolph Hughes  
Henri Schurz



## List of Participants

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This volume contains the proceedings of the International Research Conference “Probability on Algebraic and Geometric Structures”, held from June 5–7, 2014, at Southern Illinois University, Carbondale, IL, celebrating the careers of Philip Feinsilver, Salah-Eldin A. Mohammed, and Arunava Mukherjea.

These proceedings include survey papers and new research on a variety of topics such as probability measures and the behavior of stochastic processes on groups, semigroups, and Clifford algebras; algebraic methods for analyzing Markov chains and products of random matrices; stochastic integrals and stochastic ordinary, partial, and functional differential equations.

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