Recent Advances in the Geometry of Submanifolds
Dedicated to the Memory of Franki Dillen (1963–2013)

AMS Special Sessions:
Geometry of Submanifolds
October 25–26, 2014: San Francisco State University, CA

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March 14–15, 2015: Michigan State University, East Lansing, MI

Bogdan D. Suceavă
Alfonso Carriazo
Yun Myung Oh
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Editors

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Preface

About a century ago, the geometry of submanifolds gained a lot of momentum through the study of the Schl"afli's conjecture, which stated that a real analytic Riemannian manifold of dimension $n$ can be locally isometrically embedded into any real analytic Riemannian manifold of dimension $\frac{1}{2}n(n+1)$. M. Janet (1926), É. Cartan (1927) and C. Burstin (1931) made essential contributions to the understanding of the importance of the immersion problems and to a result that today bears their names. A major development for the theory was the much-celebrated Embedding Theorem, proved by John Forbes Nash, Jr. (in a series of three papers published in 1954, 1956, and 1966). Over the last several decades, many outstanding mathematicians focused their efforts on the geometry of submanifolds. Notably, Franki Dillen's work has attracted the attention of and inspired many geometers. This is why we thought it appropriate to honor his work in a volume of the American Mathematical Society's *Contemporary Mathematics* series.

Our aim was to assemble a volume that complements the existing literature with new content and new ideas that could serve as inspiration to all mathematicians working with concepts related to the geometry of submanifolds. These themes include the recent study of submanifolds in Riemannian, semi-Riemannian, Kaehlerian and contact manifolds. During the last twenty years, the study of new curvature invariants (especially the Chen curvature invariants—called by some authors $\delta$-invariants) inspired techniques that have produced new results. Some of these results have been obtained by using techniques in classical differential geometry, while others used techniques from ordinary differential equations, geometric analysis, or geometric PDEs. Of particular interest are the results focused on minimal submanifolds and their connection with various geometric functionals. Additionally, geometers have actively studied other classes of geometric objects such as totally umbilical submanifolds, ideal immersions, Lagrangian submanifolds, complex and totally real submanifolds, and submanifolds of finite type. Our research interests include the study of curvature functionals in various contexts and ambient spaces, comparison geometry, geometric PDEs, relations between curvature and topology, and other related topics. The works included in the present volume illustrate many of these ideas.

The present volume includes papers presented in two AMS Special Sessions. The first event was the *AMS Special Session on Geometry of Submanifolds*, which took place on October 25–26, 2014, at San Francisco State University, during the Western Fall Sectional Meeting (Meeting #1104). The second event was the *AMS Special Session on Recent Advances on Submanifold Geometry, Dedicated to the Memory of Franki Dillen (1963–2013)*, which took place on March 14–15, 2015, East Lansing, during the Spring Central Sectional Meeting (Meeting #1108). We
extend our thanks to David Bao, Chair of the Department of Mathematics at San Francisco State University, and to Keith Promislow, Chair of the Department of Mathematics at Michigan State University, for all the efforts that they and their collaborators invested in organizing the two conferences.

The reason that the second AMS Special Session was hosted in East Lansing is that Franki Dillen developed many research projects in collaboration with his Michigan State University co-authors, namely Bang-Yen Chen and David E. Blair. Out of these collaborations new ideas flourished. They are still inspiring many scholars working in the geometry of submanifolds. Of particular importance is the discussion of the proof of the normal scalar curvature conjecture, a question raised in 1999 by Franki Dillen and his collaborators and solved first by Zhiqin Lu, and then, independently, by Jianquan Ge and Zizhou Tang. Zhiqin Lu continued his investigation through his work with David Wenzel, which is included in this volume, along with a paper of I. Mihai investigating extensions of the same class of inequalities.

Among the most important questions still open in the geometry of submanifolds, we should mention those conjectures attributable to Bang-Yen Chen. In 1991, he formulated the biharmonic conjecture. This claims that minimal submanifolds are the only biharmonic submanifolds in Euclidean spaces. Additionally, in 1996 he conjectured that every finite type spherical hypersurface is either of 1-type or of 2-type. He also conjectured that the only finite type closed hypersurfaces of a Euclidean space are the hyperspheres. For further details about these important questions, a recent comprehensive reference is Bang-Yen Chen’s monograph Total Mean Curvature and Submanifolds of Finite Type: 2nd Edition (World Scientific, 2015). More details about these investigations are included in Ye-Lin Ou’s paper from the present volume.

A few years ago the geometry of submanifolds experienced a major development when Fernando Codá Marques and André Neves proved the classical Willmore Conjecture (originally asked in 1965). They use Almgren–Pitts min-max theory of minimal surfaces to prove that the integral of the square of the mean curvature of a torus in the three-dimensional Euclidean space is at least $2\pi^2$. It is natural to speculate as to what new classes of problems researchers in the geometry of submanifolds will focus on in subsequent decades. Are there any important questions where the new techniques developed in the larger realm of contemporary differential geometry could make a major difference? By brainstorming on the fundamental problems and exploring a large variety of questions studied in submanifold geometry, the editors hope to provide mathematicians with a working tool, not just a collection of individual contributions.

The editors would like to extend their thanks to all the scholars who participated in the two AMS Special Sessions. Their expertise and their interactions have been particularly valuable and interesting. While their papers are not included in the present volume, the contributions of Ivko Dimitric (Penn State University), Weiyong He (University of Oregon), Martin Magid (Wellesley College), Tommy Murphy (Cal State Fullerton), Mihaela Vajiac (Chapman University), Peng Wu (Cornell University), and Handan Yildirim (University of Istanbul) have been extremely valuable to and tremendously appreciated by the editors. Also, many thanks to the co-authors of the contributors to the special sessions: Nikos Georgiou (Universidade de São Paulo), Martha Patricia Dussan Angulo (Universidade de
São Paulo), Changping Wang (Normal University of Fujian), and Jingyang Zhong (University of California, Santa Cruz).

The editors of the present volume express their thanks to Michel Lapidus and Georgia Benkart, who served as AMS Secretaries in the academic year 2014–2015, when the two AMS Special Sessions were organized.

While the editors prepared the present volume, their work benefited from the outstanding support and expert consultations of several referees. Without their expertise the quality of the present volume would not be the same. Last, but not least, many thanks to Sergei Gelfand, Christine Thivierge, and Mike Saitas for their editorial guidance and support during the preparation of the present volume.

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This volume contains the proceedings of the AMS Special Session on Geometry of Submanifolds, held from October 25–26, 2014, at San Francisco State University, San Francisco, CA, and the AMS Special Session on Recent Advances in the Geometry of Submanifolds: Dedicated to the Memory of Franki Dillen (1963–2013), held from March 14–15, 2015, at Michigan State University, East Lansing, MI.

The focus of the volume is on recent studies of submanifolds of Riemannian, semi-Riemannian, Kaehlerian and contact manifolds. Some of these use techniques in classical differential geometry, while others use methods from ordinary differential equations, geometric analysis, or geometric PDEs. By brainstorming on the fundamental problems and exploring a large variety of questions studied in submanifold geometry, the editors hope to provide mathematicians with a working tool, not just a collection of individual contributions.

This volume is dedicated to the memory of Franki Dillen, whose work in submanifold theory attracted the attention of and inspired many geometers.