

CONTEMPORARY MATHEMATICS

680

Centre de Recherches Mathématiques Proceedings

Physics and Mathematics of Link Homology

Séminaire de Mathématiques Supérieures
Physics and Mathematics of Link Homology
June 24 - July 5, 2013
Centre de Recherches Mathématiques,
Université de Montréal, Québec, Canada

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Sergei Gukov
Mikhail Khovanov
Johannes Walcher
Editors



American Mathematical Society
Providence, Rhode Island

Centre de Recherches Mathématiques
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Preface

In June and July 2013, the Centre de Recherches Mathématiques in Montréal hosted a two-week long summer school on the “Physics and Mathematics of Link Homology” as part of its *Séminaire de Mathématiques Supérieures* series. Lectures were given by leading researchers working in the field of knot homology and categorification, as well as its relationship with quantum field theory and string theory. Around 90 students from across North America and Europe took advantage of the opportunity to learn about the subject from this diverse perspective.

This volume contains five expository articles written by lecturers and senior participants of that school. The list of subjects includes, from the physics side, an introduction to Chern-Simons theory and its applications by Ramadevi and V. Kumar Singh, and a review of the relation between knot homology and quantum curves by S. Gukov and I. Saberi. From the mathematical point of view, one will find B. Webster’s categorification of Reshetikhin-Turaev quantum $sl(2)$ invariants, and a survey of knot Floer homology by C. Manolescu. The relations, both proven and conjectured, between BPS algebras and geometric representation theory are reviewed in the contribution by S. Nawata and A. Oblomkov. This sample gives a faithful cross-section of the topics covered at the school. The full proceedings with video recordings of all the lectures and additional material and information are available at the url <http://www.crm.umontreal.ca/sms/2013/>.

The school was funded primarily by a consortium of North American mathematics institutes consisting of the Centre de Recherches Mathématiques, the Fields Institute, the Mathematical Sciences Research Institute, the Pacific Institute for the Mathematical Sciences, the Institut des Sciences Mathématiques, the Canadian Mathematical Society, and the Centre Interdisciplinaire de Recherche en Géométrie et Topologie. In addition, we received a contribution from the Simons Foundation for the attribution of fellowships to 20 particularly promising students. We are grateful to these institutions for their generous support.

We also wish to thank Galia Dafni for her constant encouragement and untiring support during the editorial process that led to this volume.

Heidelberg, May 31, 2016
Johannes Walcher

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The Centre de Recherches Mathématiques (CRM) was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM receives funding from the Natural Sciences and Engineering Research Council (Canada), the FRQNT (Québec), the NSF (USA), and its partner universities (Université de Montréal, McGill, UQAM, Concordia, Université Laval, Université de Sherbrooke and University of Ottawa).

Throughout recent history, the theory of knot invariants has been a fascinating melting pot of ideas and scientific cultures, blending mathematics and physics, geometry, topology and algebra, gauge theory, and quantum gravity.

The 2013 Séminaire de Mathématiques Supérieures in Montréal presented an opportunity for the next generation of scientists to learn in one place about the various perspectives on knot homology, from the mathematical background to the most recent developments, and provided an access point to the relevant parts of theoretical physics as well.

This volume presents a cross-section of topics covered at that summer school and will be a valuable resource for graduate students and researchers wishing to learn about this rapidly growing field.

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