CONTEMPORARY MATHEMATICS

701

Number Theory Related to Modular Curves Momose Memorial Volume

Seminar in Memory of Fumiyuki Momose
Barcelona-Boston-Tokyo Number Theory
Barcelona

Joan-Carles Lario
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To the memory of Fumiyuki Momose

Photo courtesy of Math. Dept., Chuo University, Tokyo, Japan

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Preface

This volume is dedicated to the memory of Fumiyuki Momose. It mostly contains articles based on talks given at a conference in his honour that was held in 2012 in Barcelona. This conference was organized by one of us (Lario) together with Jinhui Chao, Francesc Fité, Josep González Rovira and Tsutomu Sekiguchi. It was held during May 21-23 at the Facultat de Matemàtiques i Estadística of the Universitat Politècnica de Catalunya in Barcelona. There were 17 invited speakers who discussed a variety of topics in Arithmetic Geometry, and some of them have contributed articles to this volume. Many of the talks dealt with matters which were directly or indirectly inspired or motivated by the work of Momose. About 33 mathematicians from around the world participated in the conference. In addition, there were a number of friends and well-wishers of Momose who wanted to join in honouring his memory but who could not be present for the conference. They have also contributed articles for this volume.

The themes of the conference and of most of the papers in this volume are related to the arithmetic of modular curves and abelian varieties, two themes that were of great interest to Momose and which occupied most of his mathematical work. In particular, there are three articles on rational points on modular curves and on elliptic curves. The paper of Derickx, Kamienny and Mazur discusses rational families of elliptic curves over number fields that possess a rational point of order 17. The paper of Bars discusses points on modular curves that are defined over a quadratic extension of the rationals. And the paper by Arai gives an overview of his results, some joint with Momose, on points on Shimura curves defined over a number field.

There are two articles that deal with applications of arithmetic geometry to cryptography, a topic that Momose became interested in as Chuo University was home to a major Japanese Center of Excellence grant to support work in this area. One of the articles is in fact a previously unpublished work of Hashizume, Momose and Chao in which they showed how to implement the so-called Gaudry-Hess-Smart attack on a large class of elliptic curves defined over a cubic extension field of odd characteristic. The second paper by Burko shows how to do efficient point counting on certain singular superelliptic curves.

There are two articles on subjects related to automorphic forms, a topic that appeared largely indirectly in Momose's work. The paper by Gon and Oda describes explicit integral representations for Siegel-Whittaker functions on the group $Sp(2, \mathbb{R})$. The paper by Lario and Somoza discusses the Sato-Tate conjecture for a Picard curve with complex multiplication.

The paper by Castano-Bernard deals with the arithmetic of elliptic curves defined over the rational number field. He considers an elliptic curve whose L-function has a simple zero at the central critical point. For such a curve, one knows that the Heegner point construction produces a subgroup P of the Mordell-Weil group M of finite index. Castano-Bernard formulates a conjecture on the 2-part of the index $[M : P]$ in terms of the 2-part of the Shafarevich-Tate group of the elliptic curve (which in this case is known to be finite by the work of Kolyvagin and the theorems on modularity).

The paper by Ram Murty gives an overview of recent work on transcendence of special values of Dirichlet series. A second paper by Ram Murty in collaboration with Tapas Chatterjee and Siddhi Pathak discusses a criterion for Dirichlet series with periodic coefficients to vanish at the edge of the critical strip. This is in the spirit of a conjecture of Chowla.

Finally, the article of Kumar Murty deals with the construction of some abelian extensions of real quadratic fields in the spirit of work of Shimura. It uses ideas of Momose and Ribet to generalize Shimura's construction.

At the beginning of the collection, Takeshi Saito gives a brief description of Momose's mathematical work as well as a full list of Momose's publications.

We would like to thank a number of people and institutions for their effort and patience to make the conference and this volume possible. First, the authors for their generosity in contributing an article and the anonymous referees for their help in reviewing the submissions. Second, the staff of the Facultat de Matemàtiques i Estadística at UPC where the conference took place for their collaboration. Third, the staff at the AMS, especially Sergei Gelfand, Christine Thivierge, and Lauren Foster for their patience and guidance. Finally, we thank all the close friends and family of Fumiyuki Momose for their interest and help taking care of all details.

Fumiyuki Momose was a gentle and kind human being, and an excellent mathematician. We are honoured to be associated with a volume in his memory.

Joan-Carles Lario
V. Kumar Murty

The Barcelona Conference

I do not know much about Fumiyuki Momose's life. But what I do know are the true reasons why one day I decided to promote the organization of this seminar in his honour. And that is what I would like to tell you now.

I met Fumiyuki Momose in 1993 in Boston. That year I was visiting the Mathematics Department of Harvard University. After a few months there, I was asked to give a talk in the Number Theory Seminar that meets weekly every Wednesday afternoon. It seemed to me that the more natural thing to do was to explain what I had been doing during my stay in that formidable atmosphere.

When I finished my talk, there were a number of questions and comments. They were all positive and suggestive, especially one by Noam Elkies. When we went out of the room of the seminar, Fumiyuki Momose, who had been in the audience, approached me. At that time he was also visiting the department but so far we had not had any opportunity to meet. His first words were: "I did what you have told 7 years ago and it is already published. Come with me to have dinner."

That was a surprise to me! His positive and open reaction strongly contrasted with what I was used to. Instead of getting angry on his part, I saw joy and understanding. Fumiyuki Momose went even further. What I just told you happened in June 1993, shortly before the first announcement of Andrew Wiles of a proof of Fermat's Last Theorem. Three months later, back into our respective countries, I received an email where Momose invited me to spend a year at the Chuo University in Tokyo. I spent 3 months there, and I felt at home thanks to Momose.

Since that time, we had a very good relationship with various comings and goings between Barcelona and Tokyo for short stays, for his and my part. Finally, Momose had decided to come for a long sabbatical year in Barcelona. All preparations were ready; his accommodation was already set. And then he received the news about his illness and we had to abort his stay in Barcelona.

I have no doubt that his reaction to my talk in 1993 led to a deep change in the way I see things now, and in the way I understand science and life. For a long time, I had been wondering what is more important in number theory, either the theory or the numbers. Thanks to Momose, now I know the answer. The most important thing in number theory is the people.

Joan-Carles Lario

Photo circa 1995. Courtesy of Jinhui Chao

My friend, Fumiyuki Momose

I first heard of Fumiyuki Momose when I was a graduate student. At that time, I was thinking about the Tate conjecture for Abelian varieties. My advisor, John Tate, had just received a preprint from Ken Ribet in which he had proved the conjecture for Abelian varieties that are quotients of the Jacobians of modular curves. Tate gave me that paper to read and it turned out to be very important in the research that eventually formed my thesis. In that paper, Ribet mentioned that the same results had been obtained by Momose, who I later learnt was a student of Yasutaka Ihara.

That year, Takayuki Oda was visiting Harvard and was developing his theory of periods of Hilbert modular forms to prove the Tate conjecture for Hilbert modular surfaces. Oda, who I used to discuss mathematics with, told me that the following year, Momose would be visiting Harvard. I was delighted to hear this and was eagerly awaiting Momose's arrival.

When we met, I found Momose to be a very cheerful and modest human being, but with deep knowledge of arithmetic geometry. We became friends very quickly. After his thesis work, he had started to look at Mazur's work on rational points on modular curves. He was able to prove the analogue of Mazur's finiteness theorems for many curves that had not been covered in the earlier theory. He shared with me some of his preprints and the techniques I learnt there helped me in my own work. My contribution to this volume on the construction of some abelian extensions of real quadratic fields was directly influenced by Momose's work.

While Momose was very generous to everyone, he did not seem to be very attentive about his own health. He had an apendectomy soon after his arrival at Harvard. One day, I saw him in the Department limping around and he told me that he had 'escaped' from the hospital. I was amused at the time, but it was clear that he should have been resting and given himself more time for convalescing.

Over the years, we stayed in touch. When I visited Japan in 1990 for the International Congress of Mathematicians and for a satellite conference, Momose kindly took me around and showed me many wonderful places. During 2002-2006, we started meeting regularly again when Chuo University was involved with a Center of Excellence grant from the Japanese government to study number theory and cryptography. It was during this time that Momose proved some very interesting results on elliptic curves which were susceptible to the attack initially described by Frey and refined by Gaudry, Hess and Smart.

In May of 2009, our common friend Jinhui Chao of Chuo University informed me that Fumiyuki had been diagnosed with esophagus cancer and as a result had been hospitalized. He was operated on in June of that year, and he improved a

little bit but much further treatment was required. Unfortunately, his condition deteriorated rapidly soon afterwards, and he passed away on April 24, 2010. This was devastating news to those who were near him and had not expected such a rapid negative turn of events.

Tragedy struck the family once again in 2011 when Chiaki, Fumiyuki's partner of 24 years also fell ill and passed away. The Momoses are survived by their son Takaaki, whom I met at the Barcelona conference. With the passing of his parents, Takaaki was being raised by Fumiyuki's sister Mrs. Suzuki. In my conversations with him at Barcelona, Takaaki expressed an interest in mathematics and perhaps he will pursue mathematics as a career.

Fumiyuki Momose had a very generous approach to life and to mathematics. Many mathematicians, both young and old, have benefitted from that generosity. It is an honour to present this volume of mathematics dedicated to his memory.

V. Kumar Murty

Photo of Chiaki, Takaaki, and Fumiyuki Momose. Courtesy of Jinhui Chao

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Momose Memorial Volume • Lario et al., Editors

This volume contains the proceedings of the Barcelona-Boston-Tokyo Number Theory Seminar, which was held in memory of Fumiyuki Momose, a distinguished number theorist from Chuo University in Tokyo.

Momose, who was a student of Yasutaka Ihara, made important contributions to the theory of Galois representations attached to modular forms, rational points on elliptic and modular curves, modularity of some families of Abelian varieties, and applications of arithmetic geometry to cryptography.

Papers contained in this volume cover these general themes in addition to discussing Momose's contributions as well as recent work and new results.

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