

CONTEMPORARY MATHEMATICS

717

Mathematical Problems in Quantum Physics

QMATH13: Mathematical Results in Quantum Physics
October 8–11, 2016
Georgia Institute of Technology, Atlanta, Georgia

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Preface

In this volume are the proceedings of the 13th QMath conference, which was held at the Georgia Institute of Technology during October 8-11, 2016. The origins of this series date back to 1987 when Pavel Exner and Petr Šeba initiated the conference series “Mathematical Results in Quantum Theory” (or QMath). It is worth remembering that in those days Czechoslovakia, at that time the home country of Exner and Šeba, was still behind the Iron Curtain, and it took some courage to open scientific exchange between East and West.

The scientific aim of this conference series is not only to bring together people interested in the “quantum part” of mathematical physics, but also to stimulate a search for new quantum effects and a deeper understanding of quantum physics.

The proceedings can be divided into essentially two topics. One is devoted to the many-body problem, in many ways the central problem in quantum physics and condensed matter physics. The mathematical questions there range from problems in atomic physics, e.g., the difficult problem of putting a bound on the excess charge an atom can hold, to the problem of deriving effective, i.e., simpler, equations for the behavior of large systems. A relatively new problem is ‘Many-Body Localization’. Localization, i.e., the mathematical theory of insulators, is well understood for single particle models. The new problem is to understand this localization in connection with the many-body problem, and these proceedings contain some articles devoted to this very difficult subject.

The second main topic in these proceedings concerns quantum graphs. Because of the obvious connection to quantum networks, this field has received considerable attention in recent years. It also furnishes a host of new problems that allow a sharpening of the techniques used in the field of Schrödinger operators. Interesting connections appear, e.g., to the non-linear Schrödinger equation and to one-dimensional exactly solvable models.

It has been the hallmark of mathematical physics that many of its contributions do not fit neatly into traditional categories. Either new fields get created or problems from other fields of science are brought in. In this volume we have some examples of that, such as a mathematical discussion of resonances of open dimers, a problem in chemistry, and a discussion of the measurement process in quantum mechanics. Another theme is quantum information theory which, with the possibility of quantum computers, is a very active subject nowadays.

We hope with this collection to entice researchers to take up the challenges posed by some of these problems. We gratefully acknowledge partial support from National Science Foundation Grant DMS 1643086 as well as support from the American Institute of Physics, the International Union of Physics and Applied Physics, and the International Association of Mathematical Physics.

This volume contains the proceedings of the QMATH13: Mathematical Results in Quantum Physics conference, held from October 8–11, 2016, at the Georgia Institute of Technology, Atlanta, Georgia.

In recent years, a number of new frontiers have opened in mathematical physics, such as many-body localization and Schrödinger operators on graphs. There has been progress in developing mathematical techniques as well, notably in renormalization group methods and the use of Lieb–Robinson bounds in various quantum models.

The aim of this volume is to provide an overview of some of these developments. Topics include random Schrödinger operators, many-body fermionic systems, atomic systems, effective equations, and applications to quantum field theory. A number of articles are devoted to the very active area of Schrödinger operators on graphs and general spectral theory of Schrödinger operators. Some of the articles are expository and can be read by an advanced graduate student.



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