CONTEMPORARY MATHEMATICS

768

Lie Groups, Number Theory, and Vertex Algebras

Conference on
Representation Theory XVI
June 24–29, 2019
Inter-University Center
Dubrovnik, Croatia

Dražen Adamović
Andrej Dujella
Antun Milas
Pavle Pandžić
Editors

AMS American Mathematical Society

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Preface

This volume contains the proceedings of the international conference "Representation Theory XVI", held at the Inter-University Center, Dubrovnik, Croatia, June 24-29, 2019.

The volume features nineteen original research articles in representation theory divided into three thematic groups: Lie groups, number theory, and vertex operator algebras.

Representations of Lie groups, especially unitary representations, have been and continue to be an important topic in representation theory. Although a lot is known about them, the main problem of classifying unitary representations is still not completely solved, although recently Adams, van Leuven, Trapa and Vogan constructed an algorithm that can be effectively used in low-rank cases. The importance of this subject comes from applications in harmonic analysis, geometry, physics and number theory. The conference featured a number of talks by renowned experts on this subject including Jeffrey Adams, Dan Barbasch, Jing-Song Huang, Dragan Miličić, Wilfried Schmid and David Vogan.

Number theory is one of the oldest topics in mathematics. It can be divided into elementary, algebraic, analytic and computational number theory. It has important connections and applications in algebraic geometry, representation theory and cryptography. The talks in the number theory section reflect the main subjects of interest of the Croatian number theory group, such as Diophantine equations, elliptic curves and modular forms, and contributions of international experts, including our long-time collaborators Yann Bugeaud, Clemens Fuchs, Attila Pethő and Robert Tichy.

Vertex algebras provide a natural framework for studying representation theory of infinite-dimensional Lie algebras and W-algebras with applications in number theory, combinatorics and theoretical physics. Recently a great impulse for this theory was made through deep connections with the 4-dimensional SCFT in physics. In the last decade, Dubrovnik conferences on Representation Theory have attracted many leading experts on vertex algebras including Tomoyuki Arakawa, Chongying Dong, Yi-Zhi Huang, Ching Hung Lam, Haisheng Li, Victor Kac, James Lepowsky, Masahiko Miyamoto, and others. The Dubrovnik series of conferences is a central place for exchange of ideas between the Croatian vertex algebra group, which now includes many young researchers and students, and the experts in the field. This year's invited speakers include Tomoyuki Arakawa, Ching Hung Lam, Pierluigi Moseneder Frajria, Yi-Zhi Huang, Cuibo Jiang, Victor Kac, Ivan Mirković, Masahiko Miyamoto, Anne Moreau, Nils Scheithauer, and Paolo Papi.

We thank everyone who participated in the conference, those who helped plan and run the conference, and especially to the contributors and the referees. The

conference was made possible through the generous support of the QuantiXLie Center of Excellence, a project co-financed by the Croatian Government and European Union through the European Regional Development Fund - the Competitiveness and Cohesion Operational Programme (Grant KK.01.1.1.01.0004). It was also supported by the Ministry of Science and Education of the Republic of Croatia, by the Foundation of the Croatian Academy of Sciences and Arts, and by the Inter-University Centre in Dubrovnik.

Without the hard work of the editorial staff of the American Mathematical Society, this volume would not have been possible. Our thanks go to Christine M. Thivierge for her constant help.

We greatly appreciate the staff of the Inter-University Center, Dubrovnik for their efficient and dedicated work on logistics and help during the conference.

We are very grateful to Hrvoje Kraljević, Mirko Primc and David Vogan who served with us as directors of the conference. Hrvoje Kraljević also coordinated the technical part of the organization.

Dražen Adamović, Andrej Dujella
Antun Milas, Pavle Pandžić

Group photo of the participants

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Representation Theory • Adamović et al., Editors

This volume contains the proceedings of the conference on Representation Theory XVI, held from June 25–29, 2019, in Dubrovnik, Croatia.

The articles in the volume address selected aspects of representation theory of reductive Lie groups and vertex algebras, and are written by prominent experts in the field as well as junior researchers. The three main topics of these articles are Lie theory, number theory, and vertex algebras.

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