Volume 3

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Introduction to Abelian Varieties

V. Kumar Murty

American Mathematical Society

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V. Kumar Murty

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Preface

This work is based on the notes of an elementary course on *Abelian Varieties* given at Concordia University from January to May 1986. In the last few years, the notes have been distributed informally to students wishing to have an introduction to the subject. The positive response from them and from colleagues has been an encouragement to make them available on a wider basis.

Throughout, we have tried to preserve the informal style of the lectures. The aim has been to present the material in a form suitable for independent study by graduate students and by researchers in other fields who may wish an introduction to the subject.

I would like to thank Clifton Cunningham and Damien Roy for a careful reading of the manuscript and for offering many helpful suggestions and comments. I would also like to thank Abid Zaidi for typing the notes and Emile LeBlanc for help with LaTeX and other computer related problems. Finally, I thank Francis Clarke for the invitation to publish in the CRM Monograph Series, and the editorial staff of CRM for their helpfulness and efficiency.

Vijaya Kumar
Toronto
September 1992

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The book represents an introduction to the theory of abelian varieties with a view to arithmetic. The aim is to introduce some of the basics of the theory as well as some recent arithmetic applications to graduate students and researchers in other fields. The first part contains proofs of the Abel-Jacobi theorem, Riemann's relations and the Lefschetz theorem on projective embeddings over the complex numbers in the spirit of S. Lang's book *Introduction to algebraic and abelian functions*. Then the Jacobians of Fermat curves as well as some modular curves are discussed. Finally, as an application, Faltings' proof of the Mordell conjecture and its intermediate steps, the Tate conjecture and the Shafarevich conjecture, are sketched.

— ***H. Lange for MathSciNet***

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