Volume 18


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Centre de Recherches Mathématiques
Université de Montréal

# Hecke Algebras with Unequal Parameters

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https://doi.org/10.1090/crmm/018

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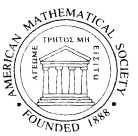


**American Mathematical Society**
Providence, Rhode Island USA

The production of this volume was supported in part by the Fonds québécois de la recherche sur la nature et les technologies (FQRNT) and the Natural Sciences and Engineering Research Council of Canada (NSERC). The work of G. Lusztig was supported by the National Science Foundation.

2000 *Mathematics Subject Classification.* Primary 20C08.

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**Library of Congress Cataloging-in-Publication Data**

Lusztig, George, 1946–
Hecke algebras with unequal parameters / George Lusztig.
p. cm. — (CRM monograph series, ISSN 1065-8599 ; v. 18)
ISBN 0-8218-3356-1 (alk. paper)
Includes bibliographical references.
1. Hecke algebras. 2. Linear algebraic groups. 3. Representations of groups. I. Title. II. Series.

QA179.L88 2003
512′.55—dc21 2002038468

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This volume was submitted to the American Mathematical Society
in camera ready form by the Centre de Recherches Mathématiques.
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10 9 8 7 6 5 4 3 2 1 08 07 06 05 04 03

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ISBN 0-8218-2877-0

9 780821 828779

CRMM/17