Volume 23

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CRM MONOGRAPH SERIES

Centre de Recherches Mathématiques
Université de Montréal

Mathematical Techniques for Analyzing Concurrent and Probabilistic Systems

J. J. M. M. Rutten
Marta Kwiatkowska
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Preface

The impact of mathematics and logic on theoretical computer science has been great from its very birth. It is perhaps less widely known among general mathematicians that mathematics has had a profound impact on computer systems. It has been about 40 years since pioneers like McCarthy, Strachey, Landin, Dijkstra, Scott and deBakker, among others, provided a firm mathematical underpinning to the study of programming languages. It is also about 30 years since Robin Milner, Tony Hoare, Carl Petri and others began looking at concurrent systems from a mathematical point of view.

At this point the theory of concurrency is a very rich subject with annual conferences and journals devoted to research in concurrency theory. The first set of lectures presents a new way of thinking about concurrency; the **coalgebraic** approach. Though it was realized implicitly from the start that coalgebra plays a key role in concurrent systems, it was only made very explicit quite recently in large part because of the work of Prof. Rutten. Roughly speaking, one can say that algebra gives the syntactic structure while a coalgebra describes the behaviour or dynamics of the system. Rutten presents an elegant "calculus of streams" for reasoning coalgebraically and shows some striking formal analogies between calculus as understood in the sense of analysis and the stream calculus.

A relatively young offshoot of concurrency theory is the theory of probabilistic, concurrent systems. The second set of lectures gives an overview of modelling and analysis of such systems. In particular the focus is on **automated tools** for reasoning about the behaviour and correctness of such systems. Prof. Kwiatkowska is one of the pioneers of this area and has been the driving force behind making the transition from theory to practice with a large active group developing implementations of the reasoning tools, carrying out case studies and, of course, enriching the theory further.

Both sets of lectures are suitable for beginning graduate students and even some advanced undergraduates. This helps fill a gap which should make it easier for graduate students to gain entry into these exciting research areas. The notes were reviewed by the editors. We wish to thank the authors for being willing to take time out of their busy research schedules to write these detailed expository notes. We hope that people will enjoy reading them as much as we enjoyed listening to the lectures in Montréal.

The two sets of notes in this volume were based on lectures delivered by Prof. Jan Rutten and Prof. Marta Kwiatkowska at a workshop on "Mathematical Techniques for Analyzing Systems" held from September 30^{th} to October 4^{th} 2002. This workshop was part of the Année Thématique of the Centre de Recherche Mathématiques devoted to theoretical computer science and we are very grateful to the CRM and its director, Jacques Hurtubise, whose generous support made

the workshop possible. We would also like to thank the staff at CRM: Jean Le-Tourneux, André Montpetit, Louise Letendre, Louis Pelletier and Josée Laferriére who were very helpful in organizing the workshop and producing the book.

The workshop was attended by about 30 participants, including nearly a dozen students, and included several other individual lectures on the theme of the workshop. We would like to thank all the participants for their stimulating comments and the exciting atmosphere at the workshop.

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The book consists of two sets of lecture notes devoted to slightly different methods of analysis of concurrent and probabilistic computational systems.

The first set of lectures develops a calculus of streams (a generalization of the set of natural numbers) based on the coinduction principle coming from the theory of coalgebras. It is now well understood that the interplay between algebra (for describing structure) and coalgebra (for describing dynamics) is crucial for understanding concurrent systems. There is a striking analogy between streams and formula calculus reminiscent to those appearing in quantum calculus. These lecture notes will appeal to anyone working in concurrency theory but also to algebraists and logicians.

The other set of lecture notes focuses on methods for automatically verifying probabilistic systems using techniques of model checking. The unique aspect of these lectures is the coverage of both theory and practice. The authors have been responsible for one of the most successful experimental systems for probabilistic model checking. These lecture notes are of interest to software engineers, real-time programmers, researchers in machine learning and numerical analysts who may well be interested in seeing how standard numerical techniques are used in a novel context.

Both sets of lectures are expository and suitable for graduate courses in theoretical computer science.

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