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Elliptic Boundary Value
Problems with Fractional
Regularity Data

The First Order Approach

Alex Amenta
Pascal Auscher

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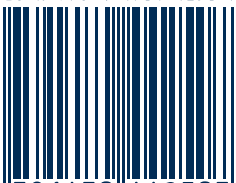
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