

Volume 2

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Centre de Recherches Mathématiques
Université de Montréal

Optimal Control via Nonsmooth Analysis

Philip D. Loewen



American Mathematical Society

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Philip D. Loewen

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Preface

For four wonderful weeks during July and August 1992, the Centre de Recherches Mathématiques of the Université de Montréal sponsored a Summer School on Control Theory. The School offered four courses—these notes grow out of the one I was fortunate enough to teach. Most of these notes were written before the lectures began, and handed out to the students to supplement the material we discussed in class. In polishing them for publication, I have tried to keep the students and their needs constantly in mind. My goal in this writeup, as in the lectures, is to build an accessible and thorough foundation describing the theory's main results, from which newcomers to the field can undertake further explorations with confidence.

Many people have influenced the contents and presentation of these notes, and I am grateful to all of them. But I would like to single out three for special thanks. First, I thank Francis Clarke, to whom many of the best ideas between these covers owe their beginnings, and whose leadership both in organizing the school and in shaping its content made the whole event such a profitable experience. Many of the exercises in these notes come from assignments Professor Clarke set for a graduate course I took in 1982-83. Second, I thank John D.L. Rowland, whose active engagement with the material before the course, during the lectures, and now during the final stages of the writeup has done much to improve the final product. Third, and most important, I thank my beloved wife, Kimberley T. Ponich Loewen, who went beyond being patient with this project to the point of actively encouraging it.

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