Optimal Control via Nonsmooth Analysis

Philip D. Loewen
Titles in This Series

Volume
2  Philip D. Loewen
   Optimal control via nonsmooth analysis
   1993

1  M. Ram Murthy, Editor
   Theta functions
   1993
This page intentionally left blank
Optimal Control via Nonsmooth Analysis
Optimal Control via Nonsmooth Analysis

Philip D. Loewen

The Centre de Recherches Mathématiques (CRM) of l'Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral and publication programs. CRM is supported by l'Université de Montréal, the Province of Québec (FCAR), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with l'Institut des Sciences Mathématiques de Montréal (ISM), whose constituent members are Concordia University, McGill University, l'Université de Montréal, l'Université du Québec à Montréal, and l'Ecole Polytechnique.
Contents

Preface ix

Chapter 1. Motivation 1
A. The Calculus of Variations 2
B. Optimal Control 12
C. Nonsmooth Analysis 25
D. Recommended Reading 28
E. Exercises 29
References 31

Chapter 2. Existence of Solutions 33
A. Review of Measure Theory 34
B. Measurable Multifunctions and Selections 34
C. Differential Inclusions 38
D. The Set of Trajectories 42
E. Existence of Solutions 49
F. Relaxation 56
G. Exercises 57
References 60

Chapter 3. Variational Principles 61
A. Introduction 61
B. The Smooth Variational Principle of Borwein and Preiss 63
C. Applications 65
D. Exercises 70
References 72

Chapter 4. The Geometry of Nonsmooth Analysis 73
A. Proximal Normals and Subgradients 74
B. The Weak Topology 78
C. Limiting Normals and Subgradients 80
D. Duality 86
E. Exercises 91
References 94

Chapter 5. Subgradient Calculus 95
A. The Main Results 95
B. Suggested Reading 106
C. Exercises 106
References 107
Chapter 6. Necessary Conditions in Dynamic Optimization

A. The Generalized Problem of Bolza 109
B. The Lipschitz Problem of Bolza 111
C. The Free-Endpoint Differential Inclusion Problem 113
D. Endpoint Constraints 120
E. The Pontryagin Maximum Principle 129
F. A Bolza Problem 131
G. Exercises 134
References 137

Chapter 7. Dynamic Programming 139

A. The Principle of Optimality 139
B. The Verification Theorem 141
C. Feedback Optimal Control 145
D. Differential Characterization of the Value Function 147
E. Exercises 151
References 153
Preface

For four wonderful weeks during July and August 1992, the Centre de Recherches Mathématiques of the Université de Montréal sponsored a Summer School on Control Theory. The School offered four courses—these notes grow out of the one I was fortunate enough to teach. Most of these notes were written before the lectures began, and handed out to the students to supplement the material we discussed in class. In polishing them for publication, I have tried to keep the students and their needs constantly in mind. My goal in this writeup, as in the lectures, is to build an accessible and thorough foundation describing the theory's main results, from which newcomers to the field can undertake further explorations with confidence.

Many people have influenced the contents and presentation of these notes, and I am grateful to all of them. But I would like to single out three for special thanks. First, I thank Francis Clarke, to whom many of the best ideas between these covers owe their beginnings, and whose leadership both in organizing the school and in shaping its content made the whole event such a profitable experience. Many of the exercises in these notes come from assignments Professor Clarke set for a graduate course I took in 1982-83. Second, I thank John D.L. Rowland, whose active engagement with the material before the course, during the lectures, and now during the final stages of the writeup has done much to improve the final product. Third, and most important, I thank my beloved wife, Kimberley T. Ponich Loewen, who went beyond being patient with this project to the point of actively encouraging it.

Philip D. Loewen
Vancouver 1992