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Centre de Recherches Mathématiques
Université de Montréal

Plates and Shells

Michel Fortin
Editor



American Mathematical Society

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Preface

The 1996 Summer Seminar of the Canadian Mathematical Society took place in Québec city from July 22 to July 27, in the building of l'École d'Architecture at the Université Laval. The scientific committee consisted of

- Michel Fortin (chair), Département de mathématiques et de statistique, Laval,
- Mario Fafard, Département de Génie civil, Laval,
- Michel Delfour, Centre de recherches mathématiques, Montréal.

The objective of this Seminar was to bring together both mathematicians and engineers interested in the theory or application of plates and shells, or more generally in the modelisation of thin structures. From this it was hoped that there could emerge a better understanding of the problem: mathematicians could become more aware of how actual engineering applications were performed and engineers would know better what information of theoretical results could provide.

Thin structures are met everywhere in applications. Their approximation by a lower-dimensional model is natural and almost compulsory. It is also full of pitfalls and some engineering catastrophes can be attributed to bad numerical schemes for shells.

From the mathematical point of view, many studies try to make clearer the convergence of full three-dimensional models to models of plates or shells. In particular, the underlying assumptions are made more explicit.

Numerical methods are the key to actual computations. Thin structures are known to need special methods, often inspired by mixed finite element methods. The goal is to avoid numerical locking while providing a good accuracy.

New applications, such as the simulation of crash in the automobile industry, bring new challenges as models and method are stretched to their extreme limit.

All these aspects are reported in these Proceedings which give a snapshot of the state of the art of a fast evolving subject.

Michel Fortin
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