Volume 23

CRM PROCEEDINGS & LECTURE NOTES

Centre de Recherches Mathématiques
Université de Montréal

Graph Colouring and Applications

Pierre Hansen
Odile Marcotte
Editors

American Mathematical Society

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The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FCAR), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal, whose constituent members are Concordia University, McGill University, the Université de Montréal, the Université du Québec à Montréal, and the Ecole Polytechnique. The CRM may be reached on the Web at www.crm.umontreal.ca.

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Preface

From May 5 to May 9, 1997, the CRM hosted a workshop on Graph Colouring and Applications. The workshop was organized by Pierre Hansen and Odile Marcotte and was jointly sponsored by CRM and GERAD (Groupe d'études et de recherche en analyse des décisions). It brought together outstanding researchers in the fields of combinatorial optimization and graph theory, and included eleven lectures by invited speakers and seventeen contributed talks.

The opening lecture was given by Paul Seymour, from Princeton University, who presented the latest and most elegant proof of the famous four-colour theorem. This proof is the result of work carried out by Paul Seymour and his collaborators, several of whom were also attending the workshop. The lecture of Professor Carsten Thomassen, from the Technical University of Denmark, had a similar topic, i.e., algorithms for colouring graphs embedded in specific surfaces. Professors Claude Berge (from Paris) and Vasek Chvátal (from Rutgers University) gave lectures on perfect graphs, a topic closely related to the colouring problem. Professor Anthony Hilton (from Reading) lectured on total colourings of graphs, and Professor Adrian Bondy (from Lyon) on the relationship between colourings and orientations of graphs.

Graph colouring has many applications, some of whom arise from practical concerns and others from the natural or social sciences. Professor Horst Sachs (from the Technical University of Ilmenau, in Germany) presented results on invariant polynomials of polyhedra and their application to chemistry. Dominique de Werra (from the École Polytechnique Fédérale de Lausanne) gave a lecture on the application of graph colouring to the problem of register allocation, which arises during the compilation of computer programs. Fred Roberts (from Rutgers University) presented a variant of graph colouring used in the modelling of social roles, and related this concept to other important concepts in the mathematical models of the social sciences.

Finally, Mike Carter (from the University of Toronto) and Bjarne Toft (from Odense University, in Denmark) lectured on applications of graph colouring to timetabling. The last main lecture was given by Bjarne Toft, who educated and entertained his audience by discussing the life and correspondence of Julius Petersen, who gave his name to the famous Petersen graph! The themes of the contributed talks were similar to those of the main talks, and the workshop was attended by 46 researchers from eight countries (Canada, United States, France, England, Germany, Denmark, Switzerland and Israel).

The articles of this volume span a wide spectrum of topics related to graph colouring: enumeration of colourings (Walsh), list-colourings (Dror et al.), total colourings (Hamilton et al.), colourings and embeddings of graphs (Collins and Hutchinson), chromatic polynomials (Arrowsmith and Essam), characteristic polynomials (Sachs), chromatic scheduling (de Werra) and graph colouring problems related to frequency assignment (Walsh, Harary and Plantholt, Marcotte and Hansen). All the articles published in this volume were refereed. The proceedings also include a list of open problems suggested by the participants.

To conclude this introduction, we would like to express our heartfelt thanks to Dr. Luc Vinet, director of the CRM, for suggesting that the workshop be held and providing financial support. We would also like to thank the referees who reviewed the articles and Dr. Yvan St-Aubin, deputy director of the CRM, who took care of the refereeing process for our own article. Finally we thank Mr. Louis Pelletier for taking care of all aspects concerning the organization of the workshop.

Pierre Hansen
Odile Marcotte
Organizers of the workshop and editors of the proceedings

Montréal, July 1999

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Open Problems

PROBLEM 1 (Sylvain Gravier). Let $c(G)$ and $\chi(G)$ be the clique number and the chromatic number of a graph G. If $c(G) < \chi(G)$ does there exist an integer n such that the join of G with a clique on n vertices is such that $c(G+K_n) = \chi(G)+n$?

PROBLEM 2 (Frank Harary). Consider the following two-person game on a 8×8 chessboard where players A and B both have 4 queens: A puts down a queen, B puts down a queen in an independent position (i.e., not taken by the queen of A), A puts down a second queen in an independent position (i.e., not taken by either of the 2 queens already on the board), etc. The last player to put down a queen wins. *Claim*: A can always win.

PROBLEM 3 (Michael Molloy). What is the maximum value of t for which every Δ-regular simple graph has a proper $(\Delta + 1)$-colouring where every vertex has at least t different colours appearing in its neighborhood?

PROBLEM 4 (Horst Sachs). Consider 3 strings of unit length, knotted in some way such that there is no knot between all 3 strings and the set of strings is connected. Given a pair of scissors one may cut a string, but not a knot. *Claim*: one can always create a string of length > 1.

PROBLEM 5 (Horst Sachs). If G is a cubic planar graph with n vertices and a 2-factor which is a union of 2 circuits, then G is edge 3-colourable. This is a special case of the 4-colour theorem; find a shorter proof.

PROBLEM 6 (Paul Seymour). If G is loopless and has no K_{n+1} minor and no induced odd circuit of length ≥ 5 or its complement, then G is n-colourable.

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Errata

Horst Sachs, *Characteristic polynomials in the theory of polyhedra*, in Graph Colouring and Applications (P. Hansen and O. Marcotte, eds.), CRM Proceedings & Lecture Notes, vol. 23, Amer. Math. Soc., Providence, RI, 1999, pp. 111–126.

Page 118, Line 13: Change "Lemma 10" to "Lemma 2".

Page 121: Figure 5 and Table 4 are correct as printed. Replace the text on this page by the following:

8. The Character Schemes of the Connected Subgraphs

Let G be a plane undirected graph, $\omega \in \Omega(\mathrm{G})$ and $\chi_\omega = \chi$. According to Proposition 2, the corresponding skew characteristic polynomial is

$$\widetilde{f}_\chi(\mathrm{G}; x) = \widetilde{f}(\mathrm{G}^\omega; x) = x^r(x^{2s} + a_2x^{2s-2} + \cdots + a_{2s}).$$

By a classical theorem, the coefficient $a_{2\sigma}$ equals the sum of all principal minors of order 2σ of $\widetilde{\mathbf{A}}(\mathrm{G}^\omega)$. These minors are the determinants of the $\binom{v}{2\sigma}$ skew adjacency matrices of the induced directed subgraphs on 2σ vertices of G^ω. If H^ω is such a subgraph that is not connected, then the corresponding minor is the product of those minors that correspond to the components of H^ω. Therefore, it is desirable to know more about the character schemes of the connected subgraphs of G^ω.

THEOREM 2. *Let* G *be a plane undirected graph and* $\widehat{\mathrm{G}}$ *a (not necessarily induced) connected subgraph of* G. *For any face* $\widetilde{F}$ *of* $\widehat{\mathrm{G}}$, *let* $\mathbb{F}^*(\widetilde{F})$ *and* $v(\widetilde{F})$ *denote the set of those faces of* G *that are contained in* $\widetilde{F}$, *and the number of those vertices of* G *that lie in the interior of* $\widetilde{F}$, *respectively. Then, for any orientation* ω *of* G

Page 123: The "dodecahedron" line of Table 5, should appear as follows:

dodecahedron	$(x+2)^4 \times$ $x^4(x-1)^5 \times$ $(x-3)(x^2-5)^3$	$x(x-2)^5 \times$ $(x-3)^4(x-5)^4 \times$ $(x^2-6x+4)^3$	$(x^2+6)^4(x^2+1)^6$

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