Volume 25

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CRM PROCEEDINGS & LECTURE NOTES

Centre de Recherches Mathématiques
Université de Montréal

SIDE III—Symmetries and Integrability of Difference Equations

Decio Levi
Orlando Ragnisco
Editors

American Mathematical Society

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Decio Levi
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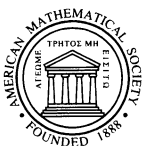
The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FCAR), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal, whose constituent members are Concordia University, McGill University, the Université de Montréal, the Université du Québec à Montréal, and the Ecole Polytechnique. The CRM may be reached on the Web at www.crm.umontreal.ca.



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Preface

The third meeting on "Symmetries and Integrability of Difference Equations" (SIDE III) was held in Sabaudia, a beautiful natural resort by the sea about 100 km south of Rome, Italy, from May the 16^{th} till May the 22^{nd}, 1998. This series of international meetings started in 1994: SIDE I was held in Estérel, Québec (near Montréal, Canada) and was organized under the auspices of the CRM (Centre de Recherches Mathématiques) of the Université de Montréal. A second meeting took place in 1996 at the University of Kent in Canterbury (UK). Information on the first SIDE meeting has been reported in *Symmetries and Integrability of Difference Equations*, edited by D. Levi, L. Vinet and P. Winternitz, AMS, 1996, and on the second in *Symmetries and Integrability of Difference Equations*, edited by P. A. Clarkson and F. W. Nijhoff, C.U.P., 1999. These meetings are intended to bring together specialists from various disciplines, all working or using methods from discrete integrable systems, i.e., systems that can be described by ordinary or partial difference equations and that allow for exact methods for their solutions. This domain forms the core of a great variety of fields, including classical and quantum physics, numerical analysis, discrete geometry, computer science, mathematical biology, economics, and so on. The subject of Discrete Systems and, in particular, of Discrete Integrable Systems has undergone a remarkable development in recent years: much progress has been made in the comprehension of the related mathematical structures. Moreover, among physicists and applied scientists in general there has been an increasing interest in discrete models, ranging from partial to ordinary difference equations to cellular automata. Accordingly, the Sabaudia meeting was pretty interdisciplinary in nature, and so is the present volume, which collects the lectures and talks therein delivered. We hope that it will provide a clear picture of the state of the art and give some insight into the most advanced research topics in the area.

The meeting attracted 72 participants from 19 countries (ordered according to the number of participants: Italy (13), Japan (10), Poland, Russia and Spain (6), Australia and Germany (4), Canada, France, Mexico, U.K and U.S.A. (3), Belgium (2), Finland, Greece, India, Romania, Sweden and Ukraine (1). Forty-five talks were delivered (among them, 12 1-hour "review lectures") and 15 posters were presented. A large part of their content can be found in this volume, which includes contributions, covering the following interconnected and even overlapping subjects:

- *Lie-type symmetries of differential-difference and difference-difference equations*: M. L. Gandarias, D. Gomez-Ullate, R. H. Heredero, L. Martina, L. M. Nieto, M. Sentihilvelan;
- *Integrable differential-difference equations*: M. Alber, M. Blaszak, C. Brezinski, A. Cârstea, I. Habibullin, B. Prinari;

• *Fully discrete integrable equations: from multidimensional lattices to cellular automata*: Y. Fedorov, R. Hirota, A. Hone, K. Kashiwara, K. Maruno, Y. Ohta, V. Verbus;

• *Discrete geometry and integrability*: L. Bogdanov, A. Doliwa, B. Konopelchenko, P. M. Santini. W. Schieff;

• *Continuous and discrete Painlevé equations and related problems*: P. G. Estevez, N. Joshi, N. Kudriashov, S. Lafortune, V. Papageorgiou, R. Quispel, A. Ramani;

• *Isomonodromy, spectral theory and related problems*: C. Cresswell, V. Enolskii, J. Harnad, W. Lay, V. Vereschagin;

• *Discrete dynamics: integrability and chaos*: J. Hietarinta, C. Viallet;

• *Difference and q-difference equations, special functions and orthogonal polynomials*: N. Atakashiev, G. Dattoli, V. Spiridonov, A. Turbiner, W. Van Assche;

• *Difference equations and quantum groups*: T. Miwa.

The success of the meeting has been confirmed by the unanimous decision to organize a SIDE IV workshop, which is planned to take place in Japan in fall 2000.

D. Levi
O. Ragnisco

Montréal, February 2000

SIDE III—Symmetries and Integrability of Difference Equations

Decio Levi and Orlando Ragnisco, Editors

This volume contains the proceedings of the third meeting on "Symmetries and Integrability of Difference Equations" (SIDE III). The collection includes original results not published elsewhere and articles that give a rigorous but concise overview of their subject, and provides a complete description of the state of the art. Research in the field of difference equations—often referred to more generally as discrete systems—has undergone impressive development in recent years. In this collection the reader finds the most important new developments in a number of areas, including: Lie-type symmetries of differential-difference and difference-difference equations, integrability of fully discrete systems such as cellular automata, the connection between integrability and discrete geometry, the isomonodromy approach to discrete spectral problems and related discrete Painlevé equations, difference and q-difference equations and orthogonal polynomials, difference equations and quantum groups, and integrability and chaos in discrete-time dynamical systems.

The proceedings will be valuable to mathematicians and theoretical physicists interested in the mathematical aspects and/or in the physical applications of discrete nonlinear dynamics, with special emphasis on the systems that can be integrated by analytic methods or at least admit special explicit solutions. The research in this volume will also be of interest to engineers working in discrete dynamics as well as to theoretical biologists and economists.

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