Volume 26

CRM

CRM PROCEEDINGS & LECTURE NOTES

Centre de Recherches Mathématiques
Université de Montréal

Integrable Systems: From Classical to Quantum

Proceedings of the 38th Session of the
Séminaire de mathématiques supérieures
July 26–August 6, 1999
Montréal, Québec, Canada

J. Harnad
G. Sabidussi
P. Winternitz
Editors

American Mathematical Society

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Preface

The 1999 Séminaire de mathématiques supérieures concerned the modern theory of integrable systems, both classical and quantum. This subject has been in a remarkably active state of development throughout the past three decades, finding renewed motivation for its study in some quite surprisingly new directions. Besides the intrinsic interest in the study of integrable models of many-particle systems, spin chains, lattice and field theory models at both the classical and the quantum level, and completely solvable models in statistical mechanics, there have been new applications in relation to a number of other fields of current interest in theoretical physics and pure mathematics, such as: the Seiberg-Wittten approach to supersymmetric Yang-Mills theory, the spectral theory of random matrices, topological models of quantum gravity, conformal field theory, mirror symmetry, quantum cohomology, etc.

The series of lectures at the 1999 SMS gave a nice cross-section of the current state of work in the area of integrable systems, presented by some of the leading researchers active in this area. There were a total of 65 participants, coming from a dozen different countries in Europe, North America and the Far East. The principal lecture series each consisted of a sequence of between three and five hour-long lectures, and these were supplemented by a number of more specialized presentations by some of the participants. Three of these, given by L. Fehér, D. Korotkin and J.-M. Maillet have been included in the present volume.

The topics covered by the principal lectures in the present volume include: universality of distribution functions in random matrix theory (C. A. Tracy and H. Widom), the classical R-matrix approach to isospectral flows and their algebro-geometric linearization (J. Harnad), the extension of the latter to the elliptic R-matrix case and Hitchin systems, and the link between the latter and Calogero-Moser systems (J. Hurtubise), integrability and symmetry of the XXZ spin chain model (T. Miwa), the relation of Bäcklund transformations to the Baxter Q-operator (E. Sklyanin), characteristic systems on Poisson Lie groups (N. Reshetikhin), multi-separability and superintegrability of classical and quantum systems (W. Miller, Jr.), determinant representations of form factors (V. Korepin) and the developments in special function theory related to quantum integrable systems of Calogero-Moser type (S. Ruijsenaars). The supplementary topics include: the solution of the quantum inverse scattering problem for spin chains and its application to the computation of correlation functions (J.-M. Maillet), algebro-geometric solutions of isomonodromic deformation equations (D. Korotkin), and Poisson-Lie groupoids in relation to the chiral WZNW phase space (J. Balog, L. Fehér, and L. Palla).

Unfortunately, not every lecture sequence was available in time for inclusion, and some that are included are presented in a somewhat reduced form. Among the

excellent series which unfortunately could not be included here are the one by A. Its, on "The Riemann-Hilbert Approach to Exactly Solvable Quantum Field Theory and Statistical Mechanical Models," the one by A. Polychronakos on "Calogero-Like Systems; Physics and Mathematics" and the one by P. Winternitz, on "Integrable Systems, Symmetries and Lie Algebra Contractions".

The series that are presented in this volume in a somewhat shortened form include the ones by T. Miwa, whose original title was "Algebraic Analysis of Solvable Lattice Models"; by C. A. Tracy, whose title was "Random Matrix Models and Integrable Systems"; by V. Korepin, whose original title was "Determinant Representations for Quantum Correlation Functions for Exactly Solvable Models"; by N. Reshetikhin, whose title was "Quantization of Integrable Systems", and by J. Harnad, which originally included an additional lecture covering the transition from classical to quantum systems solvable by separation of variables techniques.

The scope and quality of the lectures that are included will, we hope, make this a useful resource for all those interested in an up-to-date introduction to and overview of many of the main currents in the theory of integrable systems at the end of the second millennium.

The Editors

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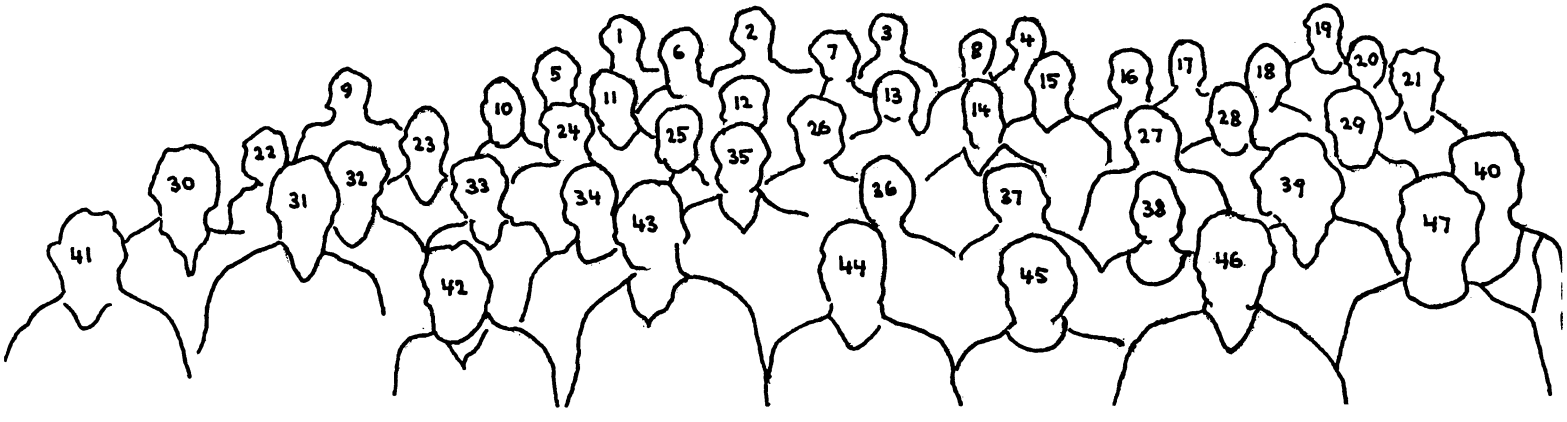
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