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Centre de Recherches Mathématiques
Université de Montréal

Topics in Probability and Lie Groups: Boundary Theory

J. C. Taylor
Editor

American Mathematical Society

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Preface

J. C. Taylor

In September 1992 I organized a CRM sponsored two week long seminar that was held on the campus of McGill Univerity. Its theme was *Topics in Probability and Lie Groups: Boundary Theory.* The seminar was largely devoted to the presentation of five mini-courses on subjects related to the theme. The lecturers and their subjects were

Persi Diaconis, *Finite Groups of Lie Type*

Steven Evans, *Local Fields, Gaussian Measures, and Brownian Motions*

Paul Gérardin, *Affine Buildings*

Yves Guivarc'h, *Boundaries and Lie Groups*

François Ledrappier, *Some Asymptotic Properties of Random Walks on Free Groups*

The initial hope for this volume was to present the lectures delivered at the seminar plus any shorter communications that the 40 or so participants desired to offer. After a year or so, it began to look as thought this goal was not going to be realised. Of the five lecturers only Steve Evans and Francois Ledrappier submitted texts of their lectures and only one participant, Alessandro Figa-Talamanca, contributed a short paper.

Steve Evans lectures are a survey of his ground breaking work on Brownian motion in the context of local fields. He determines, for example, the analogues for local fields of the classical Gausssian measures in $\mathbb{R}^n$ by starting from the observation that a random variable X is Gaussian if and only if $A \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ has the same law as $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ whenever X_1, X_2 are two independent copies of X and A is an orthogonal matrix. Mimicking this in the context of local fields leads to the analogue of a Gaussian measure.

In François Ledrappier's lectures on random walks of free groups F with a finite number of generators he examines properties of the harmonic measure at infinity, defined on the space of ends of the Cayley graph. He is particularly interested in the measure of small balls relative to the natural distance. Results are obtained when the random walk has finite support or satisfies a moment condition. The free group F is a simple model for other situations in the extensive literature that he cites.

Alessandro Figa-Talamanca's note shows how to use Gelfand pairs in connection with finite trees to compute spherical functions, as did Letac [**L**] earlier. He then uses them to compute the probability associated with a natural random walk on

the ends of a finite tree whose automorphism group is transitive on this set. This leads to a limiting problem on an infinite homogeneous tree.

As editor, with these three contributions in hand I continued to hope that further contributions would be forthcoming and thereby brought about an enormous delay in publication. For this I offer sincere apologies to the three individuals who submitted manuscripts. When I finally accepted that the original goal was not going to be realized, I had the problem of deciding what to do with the manuscripts that had been submitted. The choice was to either abandon the proposed publication or to find a reasonable way to include additional material. The second way was eventually chosen.

In the meantime, the subject matter of the seminar evolved. The elementary introduction to affine buildings that did not materialise at the seminar, was well presented, in a later seminar run by Adam Korányi in 1995, by two talks, one by Donald Cartwright on *A Brief Introduction to Buildings* and the other by Tim Steger on *Local Fields and Buildings.* They were published in volume **206** of Contemporary Mathematics edited by Adam Korányi in 1997 [**K**]

In addition, Guivarc'h, Ji, and myself became involved in writing a book entitled *Compactifications of Symmetric Spaces* [**G**] following the publication by Olshanetsky in [**O2**] of proofs for his results on Martin compactifications announced in 1969 [**O1**]. While this project of Guivarc'h, Ji, and myself ended up by providing the answer as to how to complete this volume of the CRM proceedings, it also had the effect that publication was held back an excessively long time. In particular, my own contribution took much longer to complete than anticipated.

Each of the three authors ended up providing contributions:

Yves Guivarc'h, *Compactifications of Symmetric Spaces and Positive Eigenfunctions of the Laplacian*

J.-P. Anker and L. Ji, *Heat Kernel and Green Function Estimates on Noncompact Symmetric Spaces.* II

J. C. Taylor, *The Martin Compactification Associated with a Second Order Strictly Elliptic Partial Differential Operator on a Manifold.*

These three contributions are, in effect, appendices to the book we wrote on compactifications. The first, due to Guivarc'h, presents his more algebraic point of view on the material of our book (an earlier version was published as a seminar at the Université de Rennes [**G**] in 1994), in which the geometric aspects of the earlier chapters of the book are omitted. His approach to this material will no doubt be of especial use to a certain class of readers of our book.

The second contribution is a simpler account of the work of Anker and Ji [**A**] that was used in the book to handle the behaviour of the Green function along the walls of the Weyl chamber when the eigenvalue λ is strictly less than λ_0, the bottom of the L^2-spectrum. By using a parabolic Harnack inequality of Li and Yau [**LY**] they are able to obtain bounds on the Green function along the walls from bounds in the interior.

Finally, in the third article, full details are provided about the Martin compactification for diffusions on manifolds. They were not given in the book and are mainly known to specialists in axiomatic potential theory. In fact this third contribution is an attempt to present the subject of the Martin compactification for solutions of differential equations in a self-contained way, assuming an understanding of the classical results about second order partial differential equations that are to be found in Gilbarg and Trudinger [**GT**], and without using axiomatic potential

theory. The main novelty here is the use of distribution theory to prove the Riesz representation theorem following the ideas of Brelot in his CDU notes [**B**] of 1969.

The book itself is closely related to the theme of the seminar and was written so as to prepare the way for a general discussion of boundaries for affine buildings, one of the intended, but unrealized, themes of the seminar. A certain amount of work has been already done in this direction, see for example the paper by Cartwright [**C**] and the references therein, but more remains to be done.

It remains to thank the CRM, and the AMS, for their patience in waiting for the completion of this volume and, once again, to apologize to the early contributors, and also to the seminar participants, for the excessive publication delay.

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