Volume 29

CRM
PROCEEDINGS &
LECTURE NOTES

Centre de Recherches Mathématiques
Université de Montréal

Bäcklund and Darboux Transformations. The Geometry of Solitons

AARMS-CRM Workshop
June 4–9, 1999
Halifax, N.S., Canada

Alan Coley
Decio Levi
Robert Milson
Colin Rogers
Pavel Winternitz
Editors

American Mathematical Society

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Preface

This meeting took place in Halifax, Nova Scotia, Canada between June 4$^{\text{th}}$ and June 9$^{\text{th}}$, 1999. It brought together some sixty scientists from twenty countries. The program of the Workshop consisted of two parts:

- Two preliminary days of invited lectures, the latter being each of two hours duration and intended as an introduction for non-specialists to the subject of Darboux and Bäcklund transformations.
- The main body of technical lectures for those specialists in soliton theory with a research interest in its geometric underpinnings with emphasis on the role of Bäcklund and Darboux transformations.

Bäcklund and Darboux transformations have a long history and both owe their origin to geometric investigations carried out in the 19$^{\text{th}}$ century. Thus, Bäcklund transformations initially arose in connection with the construction of pseudospherical surfaces. The Gauss–Weingarten system for the latter, in asymptotic coordinates immediately provides a contact with soliton theory in that it yields, via its compatibility conditions, the integrable sine-Gordon equation. The latter was obtained originally in this geometric context by Edmond Bour in 1862. The following year, Julius Weingarten set down another integrable equation, namely the hyperbolic sinh–Gordon equation in his treatise on surfaces of constant mean curvature.

Both pseudospherical surfaces and surfaces of constant mean curvature are included in a more general category of surfaces that had been introduced in 1861 by Weingarten and which are defined by the requirement that the principal curvatures are functionally dependent. Ribacour in 1872 studied such Weingarten or so-called W surfaces and obtained results which are akin to those derived later by Bianchi in 1879 in his habilitation thesis. Therein, what has become known as the Bianchi transformation for the construction of pseudospherical surfaces was presented. The celebrated Bäcklund transformation for pseudospherical surfaces was subsequently derived in 1883 and constitutes a conjugation of the parameter-independent Bianchi transformation and a Lie symmetry which serves to intrude a vital "Bäcklund" parameter. The latter proved essential to the next major development which took place in 1892. In that year, under the title *Sulla Transformatione di Bäcklund per le Superficie Pseudosferiche* the Italian geometer Luigi Bianchi, in a masterly breakthrough, established that the Bäcklund transformation at the level of the sine-Gordon equation admits a commutative property. A consequence of this is a nonlinear superposition principle embodied in what Bianchi termed a permutability theorem. The evidence that Bianchi's permutability theorem has important application in nonlinear physics had to await the work of Seeger et al. in 1953 on crystal dislocations. Therein, in the context of Frenkel and Kontorova's dislocation theory of 1938, the superposition of so-called "eigenmotions" was obtained via the permutability theorem. Indeed, the interaction of what today is called a breather with a kink-type dislocation was both described analytically by means of the permutability theorem and displayed graphically. The typical solitonic features to be later discovered numerically by Zabusky and Kruskal in 1965 for the Korteweg–de Vries equation, namely preservation of velocity and shape following interaction, as well as the concomitant phase shift were all derived via the permutability theorem for the sine-Gordon equation in this remarkable work.

In 1967, Lamb obtained the classical sine-Gordon equation in an analysis of the propagation of ultrashort light pulses. He had become aware of the earlier work of Seeger et al. and exploited the permutability theorem associated with the Bäcklund transformation to generate an analytic expression for pulse decomposition corresponding to the two-soliton solution. Later in 1971, he used the permutability theorem to analyse the decomposition of $2N\pi$ light pulses into N stable 2π pulses. The experimental evidence for such a decomposition behaviour had been provided by Gibbs and Slusher in 1970 who recorded the decomposition of a 6π pulse into there 2π pulsed in a *Rb* vapour. In the same year, Scott had noted how the permutability theorem may be exploited in the study of long Josephson junctions.

It was in 1895, that two Dutch mathematicians Korteweg and de Vries had set down the nonlinear evolution equation which now bears their name and which models long wave propagation in a rectangular channel. However, it is less well-known that what is now called the Korteweg–de Vries (KdV) equation had, in fact, been set down earlier by Boussinesq in his memoir of 1877 entitled *Essai sur la Théorie des Eaux Courantes.* Indeed, a pair of equations equivalent to the KdV equation appeared as early as 1871 in two papers by Boussinesq devoted to the study of wave propagation in rectangular channels. The KdV equation admits a simple wave solution which at last provided a theoretical confirmation of the existence of the "large wave of translation" observed in 1834 by the Scottish engineer John Scott Russell on the Union Canal near Edinburgh. In 1973, Wahlquist and Estabrook demonstrated that the KdV equation, like the sine-Gordon equation, admits invariance under a Bäcklund-type transformation and, moreover, possesses an associated permutability theorem which was used iteratively to construct multi-soliton solutions. Interestingly, this nonlinear superposition principle arises elsewhere "mutatis mutandis" as the ε-algorithm of numerical analysis.

In 1974, a Bäcklund transformation for the nonlinear Schrödinger (NLS) equation was constructed by Lamb via a classical method developed by Clairin in 1910. It may also be generated in a purely geometric manner via an intrinsic formalism used in kinematic studies in hydrodynamics. A permutability theorem can again be constructed by means of the Bäcklund transformation. The NLS equation has its roots in an analysis published in 1905 by da Rios, a student of Levi Civita, of the three-dimensional motion of an isolated non-stretching vortex filament in an unbounded, inviscid fluid. Thus, da Rios obtained a pair of coupled nonlinear equations for the temporal evolution of the curvature and torsion of the vortex filament subsequently to be rediscovered by Betchov in 1965. It is these nonlinear equations that were composed by Hasimoto in 1972 to produce the NLS equation. The latter, seems to have been first set down explicitly by Kelley and Talanov in 1965 in independent studies of the self-focusing of optical beams in nonlinear Kerr media. In geometric terms, the work of Hasimoto showed that the NLS equation is generated by the evolution of an inextensible curve moving in space with local normal velocity in the direction of the binormal and with velocity magnitude proportional to the local curvature.

In 1973, the celebrated AKNS spectral scheme was introduced by Ablowitz et al. A broad spectrum of $1+1$-dimensional nonlinear evolution equations amenable to the Inverse Scattering Transform were encapsulated thereby as compatibility conditions. The linear structure of the AKNS system permits the application in soliton theory of another important class of transformations with their origin in the 19^{th} Century, namely Darboux transformations. The latter arose in a studies by Darboux in 1882 both with regard to Sturm–Liouville problems and in the theory of surfaces having spherical representation. In actuality Darboux's classical transformation is but a special case of a result due to Moutard as earlier reported by the geometers Bertrand and Bonnet in Comptes Rendus in 1870 but only published by Moutard in 1878.

Iterated Darboux transformations were subsequently later constructed by Crum in 1955 in connection with connected Sturm–Liouville problems. Twenty years later, the Crum transformation was taken up by Wadati et al. and used to generate multi-soliton solutions of integrable equations associated with the AKNS system. In geometric terms, these iterated versions of Darboux transformations arise in the classical theory of surfaces as Levy sequences and are described in Eisenhart's *Transformations of Surfaces.* In a similar manner, the binary Darboux transformation of soliton theory is nothing but the fundamental transformation (F) of conjugate nets again discussed extensively in the treatise of Eisenhart.

By 1974, the Bäcklund transformations for the canonical $1+1$-dimensional soliton equations, namely the sine-Gordon, KdV and NLS equations were all in place. Moreover the derivation of the AKNS scheme had opened the way to the application of the classical Darboux transformation and its variants to the linear representations generic to solitonic equations. Thus it was timely that in 1974 a National Science Foundation meeting was convened at Vanderbilt University in the USA to assess the status and potential role of Bäcklund and associated transformations in soliton theory.

In the quarter of a century that has passed since that Meeting it has become evident that the privileged classes of surfaces that admit Bäcklund–Darboux transformations are intrinsically tied to the privileged classes of nonlinear equations that admit solitonic phenomena. It is this remarkable fact that prompted the organisation of the present international meeting. The title and scope of the Workshop was motivated by the geometric origins of Bäcklund and Darboux transformations and their subsequent important applications in soliton theory. The main objective of the Workshop was to bring together in an active forum, leading international experts in the areas of Bäcklund and Darboux transformations and the underlying geometry of soliton theory to discuss modern developments but set in an historical context. The latter aspect was regarded as of particular importance since it has only recently become apparent that many other classes of surfaces that arise in classical studies, such as, the isothermic system and associated Calapso equation, the Demoulin system of projective geometry and the Lamé system are solitonic in nature. Indeed, Bianchi diagrams associated with classical permutability theorems for such systems can be interpreted as quadrilaterals on integrable discrete surfaces.

The broad scope of the lectures presented at the Workshop bears eloquent testament to the vigour of the subject. It supports the underlying conviction that

Bäcklund and Darboux transformations and the study of the surfaces, both continuous and discrete, that admit such transformations have a permanent place in that important area of nonlinear physics that has become known as soliton theory.

The proceedings follow the structure of the meeting itself. Part 1 contains four of the six lecture series presented during the introductory part of the workshop. Part 2 contains 36 of the 47 contributions presented at the AARMS-CRM workshop. All contributions were refereed. For multiauthored contributions the name of the author who presented the talk in printed in italics. I some cases two speakers combined their contributions into one article. Then both names appear in italics.

A. Coley, D. Levi
R. Milson, C. Rogers
P. Winternitz

Montréal, 2001

Albert Victor Bäcklund

Born: Väsby, Sweden 1845
Died: Lund, Sweden 1922

Bäcklund received his tertiary education at the University of Lund. In 1864, he was appointed as an assistant at the Astronomical Observatory where he became a student of Professor Axel Möller. In 1868, Bäcklund received his Doktor Philosophiae for a thesis concerning the measurement of latitude from astronomical observations. Bäcklund thereafter turned to geometry and, in particular, to the work of the Norwegian mathematician Sophus Lie.

In 1874, Bäcklund was awarded a government travel scholarship to pursue his studies on the Continent for six months. He spent most of his time in Leipzig and Erlangen where he met both Klein and Lindemann. Ideas Bäcklund gained in this period inspired his later work on geometry on what have come to be known as Bäcklund transformations.

In 1878, Bäcklund was appointed to the new Extraordinary Chair in Mechanics and Mathematical Physics at Lund. He was elected a Fellow of the Royal Swedish Academy of Sciences (KVA) in 1888. In 1897, Bäcklund was awarded the Chair in Physics at the University of Lund where he was subsequently Rector during the period 1907–1909. Bäcklund retired in 1910 but continued his scholarly research until his death.

Bäcklund is usually associated with the type of transformation of surfaces that bears his name, the extensions of which have had major impact in soliton theory. Bäcklund's geometric work in this area was originally motivated by an attempt to extend Lie's theory of contact transformations. Bäcklund also made a significant incursion into the theory of characteristics which originated in the work of Monge and Ampère. Indeed, Bäcklund was regarded by both Goursat and Hadamard as the founder of the modern theory of characteristics.

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Gaston Darboux

Born: Nîmes, France 1842
Died: Paris, France 1917

Darboux entered the École Normale Supérieure in 1861 and completed his doctoral thesis sur "Les surfaces orthogonales" (1866). Between 1873 and 1878 he was suppléant to Liouville in the chair of rational mechanics at the Sorbonne and in 1880 succeeded to a chair of higher geometry previously held there by Chasles and retained it until his death. In 1884, he became a member of the Académie des Sciences. In 1902 he was elected a Fellow of the Royal Society of London and was awarded its Sylvester Medal in 1916.

Darboux's primary contributions were to geometry although he also made important incursions into analysis. He produced extensive works on orthogonal systems of surfaces, notably his "Leçons sur la théorie générale des surfaces", vol. I–IV (1887–1896) and the "Leçons sur les systèmes orthogonaux et les coordonnées curvilignes" (1898). The latter contains results on cyclides relevant to modern soliton theory.

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Bäcklund and Darboux Transformations. The Geometry of Solitons

Alan Coley, Decio Levi, Robert Milson, Colin Rogers, and
Pavel Winternitz, Editors

This book is devoted to a classical topic that has undergone rapid and fruitful development over the past 25 years, namely Bäcklund and Darboux transformations and their applications in the theory of integrable systems, also known as soliton theory.

The book consists of two parts. The first is a series of introductory pedagogical lectures presented by leading experts in the field. They are devoted respectively to Bäcklund transformations of Painlevé equations, to the dressing method and Bäcklund and Darboux transformations, and to the classical geometry of Bäcklund transformations and their applications to soliton theory. The second part contains original contributions that represent new developments in the theory and applications of these transformations.

Both the introductory lectures and the original talks were presented at an International Workshop that took place in Halifax, Nova Scotia (Canada). This volume covers virtually all recent developments in the theory and applications of Bäcklund and Darboux transformations.

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