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CRM PROCEEDINGS & LECTURE NOTES

Centre de Recherches Mathématiques
Université de Montréal

Symmetry in Physics

In Memory of Robert T. Sharp

P. Winternitz
J. Harnad
C. S. Lam
J. Patera
Editors



American Mathematical Society

Symmetry in Physics



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Workshop on Symmetry in Physics

in memory of

Robert T. Sharp

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The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FCAR), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal, whose constituent members are Concordia University, McGill University, the Université de Montréal, the Université du Québec à Montréal, and the Ecole Polytechnique. The CRM may be reached on the Web at www.crm.umontreal.ca.



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Preface

The Workshop on Symmetries in Physics organized at the Centre de recherches mathématiques, Université de Montréal, September 12–14, 2002, and these Proceedings are dedicated to our colleague and friend Robert T. Sharp. He passed away on October 1, 2001, after a prolonged illness. He had been associated with the Centre de recherches mathématiques, while a professor in the Physics Department at McGill University, virtually since the CRM was created in 1969. Professor Robert T. Sharp—Bob to his friends, colleagues and students—started the tradition of mathematical physics, in particular the use of algebraic and group theoretical methods, in Montréal. We started to collaborate with him in 1970 (J. P.) and 1972 (P. W.), and formed the first team in mathematical physics in Québec.

A special issue of the *Canadian Journal of Physics* was dedicated to Bob Sharp in 1994 on the occasion of his 70th birthday. The idea of putting together an issue of the *Canadian Journal of Physics* celebrating an anniversary of Bob Sharp came to us rather naturally. We felt that Bob's qualities as a teacher, scientist, and friend, although well appreciated by those who came to know him personally in Canada and abroad, deserved to be recognized more generally and made into a more visible example for those who might otherwise be prevented from seeing them by Bob's modesty and unassuming manner.

Having missed the World War II European theatre by weeks, the young, freshly trained Canadian fighter pilot Robert T. Sharp immediately volunteered for the war in the Pacific. Even that war ended before he could reach it, leaving him stranded with his Corsair squadron in England. Returning to Canada, he would consider nothing other than a similarly gallant and thrilling profession: becoming a physicist.

After a short spell on the faculty of the Physics Department of the University of Alberta in Edmonton, the former fighter pilot became physics professor at McGill University in 1953, where he had graduated under Phil Wallace's direction just two years before. At McGill, he continued for several years to be part of the Montreal Auxiliary Squadron and flew Vampire jet fighter planes.

During the last forty years, generations of physicists went through his classes. We know several who gave Bob Sharp the highest marks a teacher may desire: those colleagues who decided in his undergraduate class to become physicists.

For young students of physics the most memorable moment in their career is the first scientific publication, which separates student years from those of an active scientist. Their fellowships, future jobs, and, most importantly, their self-esteem largely depend on that publication. If ever the granting agencies in Canada required lists of publications of students one had supervised, that of Bob Sharp would be one of the most distinguished in Canada. Having always given them specific, well-defined, and difficult problems, and having provided an outstanding example by his

personal enthusiasm, intellectual integrity, and often very substantial help in the course of the work, Bob has formed several dozen young Canadian intellectuals.

For anyone who knew Bob well, it was clear that his research was always a large part of his life. Always motivated by physics, his interests often led him to rather mathematical problems, with symmetry properties playing a prominent role.

Bob Sharp started his scientific career very successfully as a nuclear and elementary-particle physicist. For instance, an early short paper of his, published in 1953 jointly with G. K. Horton, is still very much alive. It has received 146 citations since 1982, 20 of them in 2002. Early in his career Bob became interested in the applications of group theory in physics and became one of the pioneers of this type of research in Canada. He developed a simple method for calculating Clebsch–Gordan coefficients and applied it, together with his collaborators and students, to the group $SU(3)$ shortly after this group was proposed by M. Gell-Mann and Y. Ne’eman as the group underlying the classification of hadrons. One of the problems faced by physicists when applying Lie group theory is the appropriate choice of the representation basis. Often, the physically interesting basis is not sufficiently specified. Some quantum numbers are missing, the so-called missing label problem. Bob solved this problem in a highly original way for several important groups: $SU(3)$, in some cases $SU(n)$ for any n , $O(4)$, and $O(5)$.

Since the early 1970s Bob’s interests and contributions diversified. He started working on applications of noncompact groups and contributed to the development of methods for classifying their subgroups. Some of his later work was on Kac–Moody algebras and their representations and on graded contractions of Lie algebras and Kac–Moody algebras.

Probably his most important and lasting contribution is his series of articles on generating functions in group representation theory. This is a topic that started in the last century with the work of Cayley and Sylvester, followed by many others, the best known being Poincaré and Molien. For the first half of this century it remained almost dormant until essentially Bob, his students, and collaborators revived it. The majority of the generating functions known today were calculated in his and his co-workers’ papers. He knew more about the methods of deriving all sorts of generating functions than anyone in the world and extended their applications to many new group-theoretical problems.

Many of the participants at this workshop were Bob Sharp’s personal friends. Indeed, his kindness, fairness and simple human decency, together with his outgoing personality, gained him many friends. He was very much a family man, and many of us remember the warm atmosphere, created by Bob, his wife Kay, and their four children, Susan, Joan, Ted and Doug, in their home. We were very happy that his children could participate in this workshop, or at least in the workshop banquet.

The scope of these Proceedings reflects the influence that Bob Sharp had on physics and also the esteem in which he is held by his colleagues. The unifying theme of this volume is the consistent application of group theory to problems arising in physics and the further development of group theoretical tools for applications in physics.

Jiří Patera and Pavel Winternitz
Montréal

Bob Sharp: The Man and the Scientist

When Pavel Winternitz asked me to speak on this occasion, my first thought was to recall that I had already performed this function, at the banquet recognizing his retirement from McGill. Any of you who may have been at that event might find this story repetitious. If that is so, please bear with me. But Bob did not change in the intervening years.

I should like to remind you of events of the distant past, for Bob and I first met in 1947, when I left the Canadian nuclear energy project to become a professor at McGill. Bob, on the other hand, faced a transition from the air force to academia. But he had prepared himself well; as soon as the war ended he had taken correspondence courses in mathematics and physics in preparation for graduate studies at McGill.

I may also remind you that, up to the war, there had been very little theoretical physics in Canada,—a situation which reflected the scorn for theoreticians of the great Ernest Rutherford. Most Canadian physics departments did not recognize theorists, relegating them to mathematics departments where they were not fully accepted as real mathematicians. The “Applied Mathematics” department at Toronto, led by J. L. Synge and Leopold Infeld, straddled the fence and nurtured in the following years a slow but constant flow of theorists, mostly isolated from their faculty colleagues in experimental physics.

What has this to do with Bob? Simply that, to the best of my knowledge, Bob was the first addition to the modest body of postwar theoretical physicists, the first product of a renaissance which would grow exponentially in the following decades

Bob was also my first graduate student.

While I had done my Ph.D. in relativity with Leopold Infeld, my wartime occupation with nuclear reactors had led me down a quite different path. Graphite was an important component of these reactors, and it came in macroscopic blocks. It was essential to understand its properties. But solid state physics was in its infancy, and we had in the project no experts in the field. So I was sent off on a quick course with Neville Mott at Bristol. It was in this context that I found a Ph.D. problem for Bob.

It didn’t take long for Bob’s brilliance to become evident to me. I found that complicated calculations were no obstacle to him, nor were the underlying physical insights.

I will skip to Bob’s defense of his thesis, before a very distinguished committee. These events usually take about an hour, but in Bob’s case it brought a seemingly endless barrage of questions. When it was over, he asked me what was wrong

that elicited such a grilling. What I had to explain to him was that they were so impressed by the ease and insight of his responses that they did not want to stop enjoying it.

Once he had his degree in hand, I discovered something that I had not discerned—that there was a real and important difference in our outlook and interests. Bob told me that he did not want to continue working in solid state physics; that this was, for him, too specialized and not interesting enough mathematically. In short, I understood that he had loftier goals. It is Eugene Wigner who raised the question: Why is it that mathematics is so naturally adapted to our way of understanding the world? And I believe that this is the kind of question that intrigued Bob. At this point I touch lightly on the subject of this conference: Is not symmetry at the core of our understanding of the physical world? Emmy Noether's linking of dynamic variables to symmetries was an early example. Obviously I, as someone whose major field of interest has been in the physics of crystalline solids, caught up in the same net, and when I added strong magnetic fields, it was strengthened even more.

And as we are still struggling to reach an unequivocal understanding of quantum theory itself; the role of symmetry still haunts us. So when I speak of the divergence of the paths of Bob and me, it may be summarized in the fact that I had a fascination with the details of things, and Bob was concerned with the grand fundamental principles. This divide was, I believe, constructive because we complemented each other, linking the language of mathematics to the message.

The theoretical group started with Ted Morris, Bob and me, supplemented a bit later by Dave Jackson. By the sixties, when a perceptive Dean (Ken Hare) decided that it was time to move us to the physics department where we belonged. This move was both logical and in tune with the times, but was accepted with some misgivings by the existing physics department. By this time our group had expanded to cover a wide range of physics. A new post-war generation of theorists had been assembled, too numerous to recite here, and Bob was in the core of it.

There was one respect in which Bob stood out. In the late sixties and the early seventies there was a global student revolution, which led to a more inclusive role for students, one consequence of which was the introduction of student evaluations of all their courses and their teachers. You will remember the scheme of rating on a scale of 1 to 5. 1 for excellence, down to 5 for dismal failure. I remind you of this because Bob usually rated $1 + x$ with $x < 0.5$. While many of us struggled with scores of 2+ or 3+, Bob was easily the most popular of us with the students. This was not because his courses were easy—in a lecture he could fill the blackboard several times over with rather daunting and/or complicated formulae. The secret was that everything was precise and logical. Julian Schwinger was like that too; before lecturing he had smoothed out all unnecessary complications, but with Bob it was not calculated but a manifestation of the clarity of his mind.

It is the word logical which is the key to the phenomenon—and if there is one thing that stands out in describing Bob's special characteristics it is that he had a LOGICAL mind, not just applied to physics or even science alone but to all of life.

Since it is difficult to argue against logic, Bob was not an argumentative person, whatever the subject. But this did not conflict with firm convictions, in any context. On principles we could agree, but the difference between Bob's clear logic and my "yes, buts" was always present. Harry Lam recounted to me that Bob had rebuked

him for buying lottery tickets. Harry did not disagree with the contention that the odds were against him, but yet made a reasonable response—that the price of the tickets was utterly insignificant, and could make no impression on his life. But if he won, he could retire and live the rest of his life as a free man. This I would categorize as a “yes, but,” out of the scope of simple logic.

Sometimes Bob’s logic took an unexpected turn. He served in the Air force in the war—the mark of a good patriot. Yet when the Canadian parliament was debating the issue of a uniquely Canadian flag, with a fervor that shook the whole country and aroused bitter parliamentary debate, Bob made a radical proposal: “Wouldn’t it be nice to be the only country in the world that did not HAVE a flag.” Thinking about it afterwards, it struck me that Bob’s logic had been stronger than his emotions—that, like myself, he could find no rational case for worshiping Mr. Landry’s “piece of red cloth” or his blue *fleur de lys*. One’s debt to society needed a basis more substantial than this corruptible and (at that time) contentious symbol.

Another of Bob’s characteristics was an utter immunity to fear. Fear was simply not in his nature. That was one of the things, I suppose, which made him enjoy the perils of flying and his experience as an air force pilot during the war. He carried his love of flying into peacetime, apparently enjoying most the most daring of its challenges.

On one occasion, he asked me if I would like to fly with him at weekend. He “rented” small planes and kept his flying skills alive. On this particular occasion, when we went to the airport office to arrange the flight, he was told that there was a warning of an approaching storm. Bob’s reaction was to tell me to hurry so that we could take off before it was forbidden by the airport authorities. For Bob, flying through a storm would make the flight more exciting.

But the storm did not catch up with us so that, once in the air, Bob asked me if I would like to do a loop-the-loop. I had not envisioned this, but said to Bob “no, I think I would rather not.” But Bob’s goal was some excitement, so he tried again, asking if I would like a dive, pulling out at the last moment. Once again, I disappointed him; my enthusiasm for that maneuver was even less than for the loop. So he resigned himself to a boring afternoon, probably muttering beneath his breath, “Why did I ask this guy to come with me?”

But some time later I redeemed myself. Bob asked a student if he would like to go flying. The student acquiesced. But when it came to the acrobatics, the student, recognizing that Bob was his superior, felt compelled to agree to his aerial acrobatics. Bob did the loop, he did the dive, and the student threw up. When the flight was over, he took off and left Bob to deal with the mess. From thence on, I suspect, Bob was more cautious in choosing his passengers,

What Bob did not lack, however, was a sense of humor. At one point our group had bridge evenings, in which one rotated partners and adversaries and tallied up accumulated scores at the end. Bob had some prizes for the winners, but he added an extra one at the end for the player with the lowest score. His prize was a light bulb. This, he announced, was to bring some light into the darkness. It turned out that I was the recipient. The thought that went through my head was that I played bridge somewhat as Bob flew an airplane, but without any of that risk or skill that made the activity exciting.

In these tales, I have left out one of Bob's most distinctive features. It was almost impossible not to like Bob, and he seemed always to see only the best in others. There was an intrinsic friendliness and goodwill in his social and professional relationships; he rarely criticized others, and if he did it was always without rancor or ill will. Academic politics is often bitter and contentious, but he could rise above it. It was this that led me, on the occasion of his retirement, to describe him as an academic Mr. Chips, if any of you can remember that ancient story and movie about the teacher whom everyone revered. But Bob added an extra dimension which led me to add an additional feature, leaving the picture as that of Mr. Chips in aviator's goggles.

He was a good man, a modest man with a brilliant mind and a good friend to all those who were made better human beings by knowing him. He left his imprint on all of us.

Philip R. Wallace
Victoria

Bob Sharp: Teacher, Colleague, Friend

Perhaps a better title for this would have been “Waiting for Bob Sharp,” since my own experience involved several delays before finally succeeding in meeting him. I first learned about him—by reputation—while still an undergraduate physics student at McGill in the 1960’s. Somehow, I seem to have twice missed the opportunity of meeting someone rumored to be one of the “really good” physics professors at McGill—and who was specially interested in group theoretical methods in physics. This was the subject that I discovered with great delight in my final year of the honors physics program (the year was 1967) and was thrilled to learn about from Wigner’s book. This was a revelation, and I remember being amazed by the beauty and “magical” quality of the results that it led to. However that year, exceptionally, the course was not taught by Bob Sharp, since he was away on sabbatical leave at the University of Illinois.

That was the first missed opportunity. The next year, I happened to follow the same track, going to the University of Illinois, with the intention of doing my doctorate there. I heard of the visiting professor from McGill who had been there the previous year, and had taught everybody a great deal—and who was always spoken of with a certain degree of awe and respect, but who had gone back by then to Montréal.

In fact it was only several years later that I finally got to know Bob Sharp, and learn that the respect and warmth with which he was always spoken of had indeed a very valid basis, which included much more than his special talents as teacher or specialist in group theoretical methods in physics. This was after I had already completed my doctorate, not in Illinois, but as it turned out, through the help and kind encouragement of Professor Phil Wallace (and a generous Shell scholarship) at Oxford, under the supervision first of Roger Phillips, and then J. C. Taylor. That was in the days of transition between the declining “analyticity-bootstrap” approach to high energy theory, and the beginning of the great success of unified gauge theory. My thesis work unfortunately was still on the old tack—Regge and Lorentz poles—however, from a group theoretical viewpoint.

It was only after having spent a couple of years as an I.P.P. postdoc at Carleton in Ottawa, and then, in the rather depressed job market in theoretical high energy physics that existed at that time, finding myself back in Montréal with just a temporary CEGEP professor’s job during a particularly hard year—1975–76—that I got to know Bob Sharp. This was the year of the public service strikes in Quebec that eventually helped to bring about the downfall of the Bourassa government. After attending the necessary quota of rather rowdy union meetings, trying to

win the good-will of students for the CEGEP teachers' cause, but finding myself unable to return to the classroom, I decided to while away the time, not on the picket lines, but at the CRM. Here, to my delight, I found there was a group of genuine "mathematical physicists"—Pavel Winternitz, Jiří Patera and also Bob Sharp (who was often there working with them)—specially interested in symmetries and the "group theoretic approach." For me, this was rather a difficult period; it seemed as though any hope of a future career in research was fading in the nearly non-existent job market of the time. But, being at the CRM, even unofficially, was a help, since there seemed to be a "local fluctuation" there that was contrary to the general trend, with genuinely interesting work apparently going on.

Then a small gesture was made—by Bob Sharp—that psychologically meant a great deal to me. He came to me one day and said "since you are working here anyway, I think we should somehow make this official," and he proceeded to arrange to have a certain amount paid to me from his grant for the rest of the year, as a visiting research associate. Later, he also arranged for me to have an office at McGill. Bob didn't know me very well at the time—perhaps he had just attended one or two seminars I gave, but this positive gesture of encouragement, made very quietly, with no previous discussion—just seemingly a spontaneous decision on his part, made an enormous difference to me. Afterward, Pavel Winternitz and Jiří Patera, and later Steve Shnider, all contributed to creating a longer term research associate position for me at the CRM, which gave me the courage to give up the CEGEP job and focus fully on research for a number of years to follow. But the single instant of Bob's gesture was absolutely critical, and probably made the difference between my trying to continue, despite all, to pursue a career in research, or giving up. I suspect there are others who have been helped by him in various ways, perhaps also just when it was truly needed, and that it was often in such a quiet, almost incidental way that he intervened when he did, just doing what seemed to him as natural. I have always thought of him at least as much in terms of his spontaneous, simple, human good-will, and understanding, as his love of scientific research and the human milieu in which it is done. However I did have occasion, in the following years, through his lectures, and discussions, to learn of his very deep understanding of group theoretical methods and his own special approaches to the solution of problems in physics by group theoretical means.

Later, I also got to know him well as a friend with a very warm, good nature, and as a great canoeing excursion companion. This is something that he, and Doug, and Pavel Winternitz with his sons and friends had been doing for years before I was initiated into the experience. I won't talk about my own unusual predilections for ending up in the water, together with all the contents and supplies of whichever canoe had the misfortune to be carrying me. But I do remember one particular occasion at La Vérendrye Park when, on arriving a little late to our campsite, Bob took a misstep on disembarking and ended up completely soaked. What was striking was that what to others might have been experienced as at least a mildly unpleasant mishap did not seem to diminish his good humor in the slightest. He continued to be as good-natured, good spirited and sociable, while trying to dry himself off, together with his drenched things, as at any other time—maybe even more so. And if we look at his characteristic smile on the pictures posted around us—which is the way I always visualize him—it was not diminished in the least after this dunking (which, if I remember correctly, actually occurred twice in succession). His pleasure

in the environment and his good nature simply did not allow any diminishing of the pleasure of the moment.

He often used to keep us entertained by stories, at the campfire or elsewhere—and maybe it was from these, or perhaps on some other occasion that I formed an impression that there was one special time in his life in which he had been particularly happy. This was when he was a fighter pilot trainer during the war. One of the images on the posters shows him at this time, and it makes me think of one of the stories that he told of his experiences during that time. Probably others who are here, especially his family, will know of this story in more detail—or others from that time—but it made a particular impression on me because of the way he told it. It seems that at some point while on a flying mission, his plane stalled, and he started into a dive. He described the moment, saying he thought he had better get the engine going again—which after a time, he did succeed in doing—but without any apparent recollection of distress—as though he had just been curious about whether he would be able to do it in time or not. He did not describe it as a harrowing, frightening, or distressing experience—but rather as a challenging moment whose outcome was not at all clear, and he described it with the same good-humored smile and tone as one sees in the photograph of him as a pilot. Perhaps at the time, he didn't have quite such a smile for the whole time before he got out of the stall, but somehow, I think that it must have returned to him very quickly afterward, and the recollection of the incident just did not cause him any subsequent distress. He was certainly very happy in that period of his life—but I think that he was also a man who found reason for happiness in all the phases of his life.

That story came to mind many years later, when I came to visit him in the hospital after a multiple bypass operation, during which he was experiencing quite serious complications. He was in a pretty poor state, but I remembered that there had been many, many times in which health problems had loomed threateningly, yet he had managed somehow to get through them, in as good cheer as always, and bringing as little attention to the seriousness of the situation as he could. I couldn't help thinking “will you be able to get through this one too, and back into flight again, as you were able so many times in the past?” And it really seemed, for a short while, that perhaps he could do it. I briefly saw him, just once more, at the CRM, some time after he had been released from hospital. He seemed again in good cheer—I even remember discussing with him possible places to go that summer—in particular, a conference in Crete that seemed particularly to appeal to him. But unfortunately, it was not to be.

When thinking of Bob, however, the images that come to mind are inevitably the smiling ones, like the one on the poster where he is standing before a blackboard—which really recalls the animated spirit that he brought to his lectures, or a mental one of him at a campfire telling or hearing stories, from the recent or more distant past, and enjoying it in great spirits, or one of him at home, at Rosemère, with his family and the dogs, or—perhaps, the one which precedes what most of us, even his oldest friends here, could have been present to share with him—namely, the one in pilot's uniform. He must have valued and enjoyed life very much—he certainly gave a great deal to all those around him, and I'm sure he received much love in return, both from his family, and from all those, colleagues,

friends, students who knew him, learned from him, shared with him and benefited so greatly from the experience of having known him.

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