

Volume 39

CRM

CRM  
PROCEEDINGS &  
LECTURE NOTES

Centre de Recherches Mathématiques  
Université de Montréal

Group Theory and  
Numerical Analysis

P. Winternitz  
D. Gomez-Ullate  
A. Iserles  
D. Levi  
P. J. Olver  
R. Quispel  
P. Tempesta  
*Editors*



American Mathematical Society

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**American Mathematical Society**  
Providence, Rhode Island USA

2000 *Mathematics Subject Classification*. Primary 34–06, 35–06; Secondary 20–06, 39–06.

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**Library of Congress Cataloging-in-Publication Data**

Workshop on Group Theory and Numerical Analysis (2003 : Montréal, Québec)

Group theory and numerical analysis / P. Winternitz...[et al.], editors.

p. cm. — (CRM proceedings & lecture notes, ISSN 1065-8580; v. 39)

Includes bibliographical references.

ISBN 0-8218-3565-3 (alk. paper)

1. Group theory—Congresses. 2. Numerical analysis—Congresses. I. Winternitz, Pavel.  
II. Title III. Series.

QA174.W67 2003  
512'.2—dc22

2005048328

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This volume was submitted to the American Mathematical Society  
in camera ready form by the Centre de Recherches Mathématiques.

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10 9 8 7 6 5 4 3 2 1 10 09 08 07 06 05

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## Preface

The Workshop on Group Theory and Numerical Analysis, organized under the auspices of the Centre de Recherches Mathématiques, Université de Montréal, took place in Montréal, Québec, May 26–31, 2003.

This meeting brought together scientists working in several different but mutually related areas. The unifying theme was the application of group theory and geometrical methods to the solution of differential and difference equations. The emphasis was on the combination of analytical and numerical methods and also the use of symbolic computation.

The workshop attracted 51 participants from 9 countries: Canada (24), USA (12), UK (5), Norway (3), Mexico (2), Russia (2), Australia (1), Italy (1), and Ukraine (1). A total of 34 lectures was presented.

The 20 contributions presented in this volume cover several (mutually overlapping) lines of research:

- *Lie group integrators, geometric integrators, exponential integrators* (Talks presented by A. M. Bloch/A. Iserles, E. Celledoni, D. Lewis, E. Mansfield/G. R. W. Quispel, and B. Owren).

- *Symbolic computation and solutions of ordinary differential equations, partial differential equations and differential-difference equations* (Talks presented by D. Baldwin, W. Hereman, E. S. Cheb-Terrab, and G. Reid).

- *Symmetry preserving discretization of ordinary and partial differential equations* (Talks presented by C. Cyr-Gagnon, R. Kozlov).

- *Discrete and finite Fourier transforms and data processing* (Talks presented by A. M. Atoyan, J. Patera and K. B. Wolf).

- *Boundary layer perturbation theory and its symmetry respecting discretization* (Talk by C. M. Bender).

- *Discrete symmetries of difference and differential difference equations* (Talk by D. Levi).

- *Numerical methods for treating rapid oscillations* (Talk by A. Iserles).

- *Orthogonal polynomials related to Padé approximations* (Talk by A. Zhedanov).

- *Applications in biophysics and physics* (Talks by J. Tuszynski and W. Zakrzewski).

This volume has the character of a monograph and should represent a useful reference book for scientists working in this highly topical field.

Among the review talks presented at the Workshop, but not included in these Proceedings, we mention that at least two are available elsewhere, namely P. J. Olver's talk corresponds to the publication of P. Kim and P. J. Olver, *Geometric integration via multi-space*, Regular and Chaotic Dynamics **9** (2004), no. 3,



213–226 and P. Winternitz’s talk is included in D. Levi, P. Winternitz, Continuous Symmetries of Difference Equations, in *nlin.SI/0502004*.

The Workshop Organizing Committee consisted of

- P. Winternitz (Chair, CRM, Université de Montréal, Canada).
- D. Gomez-Ullate (Universitat Politècnica de Catalunya, Barcelona, Spain).
- A. Iserles (Cambridge University, United Kingdom).
- D. Levi (Universita’ degli Studi Roma Tre, Italy).
- P. J. Olver (University of Minnesota, Minneapolis, USA).
- R. Quispel (La Trobe University, Melbourne, Australia).
- P. Tempesta (SISSA, Trieste, Italy).

The organizers thank the CRM and NSF for financial support making the Workshop possible.

P. Winternitz  
Montréal, April 2005

## List of Participants

Ana Inés Ansaldo  
Institut universitaire de  
Gériatrie de Montréal  
ana.ines.ansaldo@umontreal.ca

Armen M. Atoyan  
Université de Montréal  
atoyan@CRM.UMontreal.CA

Douglas Baldwin  
Colorado School of Mines  
dbaldwin@mines.edu

Carl Bender  
Washington University  
cmb@wuphys.wustl.edu

Mhenni Benghorbal  
London, Ontario  
mbenghor@uwo.ca

Anthony Bloch  
University of Michigan  
abloch@umich.edu

Anne Bourlioux  
Université de Montréal  
bourliou@dms.umontreal.ca

Elena Celledoni  
NTNU  
elenac@math.ntnu.no

Edgardo S. Cheb-Terrab  
Waterloo, Ontario  
ecterrab@maplesoft.com

Jeongoo Cheh  
University of Minnesota  
cheh@math.umn.edu

Peter A. Clarkson  
University of Kent  
p.a.clarkson@ukc.ac.uk

Catherine Cyr-Gagnon  
Université de Montréal  
cyr@dms.umontreal.ca

Vladimir Dorodnitsyn  
Russian Academy of Sciences  
dorod@spp.keldysh.ru

David Gomez-Ullate  
Università di Bologna  
david.gomez-ullate@@upc.edu

Alfred Michel Grundland  
Univ. du Québec à Trois-Rivières  
grundlan@CRM.UMontreal.CA

Willy Hereman  
Colorado School of Mines  
whereman@mines.edu

Véronique Hussin  
Université de Montréal  
hussin@dms.umontreal.ca

Arieh Iserles  
University of Cambridge  
a.iserles@damtp.cam.ac.uk

Pilwon Kim  
University of Minnesota  
pwkim@math.umn.edu

Roman Kozlov  
University of Oslo  
kozlov@ifi.uio.no

Frédéric Laliberté  
McGill University  
flalib@po-box.mcgill.ca

François Lemaire University of Western Ontario lemaire@orcca.on.ca	Vadim Shapiro University of Wisconsin (Madison) vshapiro@engr.wisc.edu
Decio Levi Universita' di Roma Tre University levi@fis.uniroma3.it	Neil Stewart Université de Montréal stewart@iro.UMontreal.CA
Debra Lewis University of California lewis@math.ucsc.edu	Piergiulio Tempesta SISSA tempesta@fm.sissa.it
Simon J. Malham Heriot-Watt University simonm@ma.hw.ac.uk	Zora Thomova SUNY-Institute of Technology thomovz@sunyit.edu
Elizabeth L. Mansfield University of Kent at Canterbury E.L.mansfield@ukc.ac.uk	Paul Tupper McGill University tupper@math.mcgill.ca
Peter J. Olver University of Minnesota olver@math.umn.edu	Alexander Turbiner UNAM turbiner@nuclecu.unam.mx
Brynjulf Owren NTNU brynjulf.owren@math.ntnu.no	Jacek Tuszynski University of Alberta jtus@phys.ualberta.ca
Jiri Patera Université de Montréal patera@CRM.UMontreal.CA	Francis Valiquette Université de Montréal francis.valiquette@umontreal.ca
Alexei V. Penskoi Université de Montréal	Pavel Winternitz Université de Montréal wintern@CRM.UMontreal.CA
G. Reinout W. Quispel La Trobe University r.quispel@latrobe.edu.au	Kurt Bernardo Wolf Universidad Nacional Autonoma de Mexico bwolf@fis.unam.mx
Greg Reid University of Western Ontario reid@uwo.ca	Thomas Wolf Brock University twolf@brocku.ca
David Richter William Paterson university richter@wpunj.edu	Ravil Yamilov Russian Academy of Sciences yamilov@imat.rb.ru
Nicolas Robidoux Laurentian University nrobidoux@pims.math.ca	Wojciech J. M. Zakrzewski University of Durham w.j.Zakrzewski@durham.ac.uk
Robin Scott University of Western Ontario rscott2@uwo.ca	

Alexander Zhaliy  
Institute of Mathematics, Kiev,  
Ukraine  
zhaliy@imath.kiev.ua

Alexei Zhedanov  
Donetsk Institute for Physics and  
Technology  
zhedanov@kinetic.ac.donetsk.ua

Wenqin Zhou  
University of Western Ontario  
wzhou7@uwo.ca

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ISBN 0-8218-3565-3



9 780821 835654

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