Volume 47

CRM PROCEEDINGS & LECTURE NOTES

Centre de Recherches Mathématiques
Montréal

Groups and Symmetries

From Neolithic Scots to John McKay

John Harnad
Pavel Winternitz
Editors

American Mathematical Society

https://doi.org/10.1090/crmp/047

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The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FQRNT), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal. The CRM may be reached on the Web at www.crm.math.ca.

American Mathematical Society
Providence, Rhode Island USA

The production of this volume was supported in part by the Fonds Québécois de la Recherche sur la Nature et les Technologies (FQRNT) and the Natural Sciences and Engineering Research Council of Canada (NSERC).

2000 *Mathematics Subject Classification.* Primary 20D08; Secondary 11F03.

Library of Congress Cataloging-in-Publication Data

Groups and symmetries : from Neolithic Scots to John McKay / John Harnad, Pavel Winternitz, editors. p. cm. – (CRM proceedings & lecture notes ; v. 47)

Includes bibliographical references.

ISBN 978-0-8218-4481-6 (alk. paper)

1. Finite groups—Congresses. 2. Galois theory—Congresses. 3. Symmetry (Mathematics)—Congresses. 4. Modular functions—Congresses. 5. Geometry, Algebraic—Congresses. 6. McKay, John, 1939—Congresses. I. McKay, John, 1939- II. Harnad, J. P. (John P.), 1946- III. Winternitz, Pavel.

QA174.G755 2009
512′.23—dc22 2009010376

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This volume was submitted to the American Mathematical Society
in camera ready form by the Centre de Recherches Mathématiques.
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10 9 8 7 6 5 4 3 2 1 14 13 12 11 10 09

For Trinh

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Preface

The conference bearing the name of this volume, honoring the remarkable contributions of John McKay over four decades of research, took place at the CRM in Montréal during the three day period: April 27–29, 2007. There were a total of 46 invited participants, of whom eighteen gave one hour talks (one of them, Yu. Manin, gave two), and one (George Hart) gave a workshop session on polyhedral sculpture construction.

The talks covered a wide range of topics, with principal focus given to two of the areas in which John McKay has made pioneering contributions: "Monstrous Moonshine" and the "McKay Correspondence." The general areas included were: number theory, modular functions, group representations, Galois theory and algebraic geometry.

There were many who contributed to the success of this event, which had a great significance in marking the importance of John Mckay's ideas and honoring his contributions. Financial and organizational support for this conference was generously provided by Concordia University, the CRM and the CRM CICMA Laboratory. Above all, those without whose support and encouragement this event could not have taken place were:

(1) CRM Director François Lalonde, who assured the total support of the CRM, putting its facilities, resources and personnel fully behind its realization, and helping to communicate the value of decades of exceptional contributions by John McKay to mathematics.

(2) David Graham, who at the time was Dean of the Faculty of Arts and Science at Concordia University (and currently is Provost) and assured the enthusiastic and generous support of the Faculty, thus providing the recognition of John McKay's remarkable merits and contributions by his home university that he had long merited.

(3) Very substantial practical, material and moral support generously provided by:

- Graham Carr, Vice Dean, Faculty of Arts and Sciences, Concordia University
- Truong Vo Vanh, Associate Vice President for Research, Concordia University
- Louise Dandurand, Vice President for Research, Concordia University.

(4) Several others who communicated their enthusiastic support in letters and representations to Concordia University: Ram Murty (FRSC, Queens Univ.), Henri Darmon (FRSC, McGill Univ.), Jacques Hurtubise (FRSC, McGill Univ.), André Joyal (FRSC, UQÀM), as well as Université de Montréal Rector Luc Vinet.

Several mathematicians of international renown, including Yuri Manin, John Conway, Nicholas Katz, Igor Dolgachev, came as invited speakers and others, who were unable to attend, such as Don Zagier and Nigel Hitchin, have nevertheless

contributed to this volume. In all, there were a total of about sixty participants, from all corners of the globe (Japan, France, United States, Canada, United Kingdom, Germany, Argentina). There was also a very stimulating and entertaining presentation, in the form of a group workshop devoted to assembling a sculpture by mathematician-sculptor extraordinaire George Hart, consisting of a work of his, entitled "People," which is based on the symmetry group of the icosahedron, in which the participants were obliged, under the gentle coaxing of the sculptor, to mobilize their knowledge of the structural properties and cyclic subgroups.

The topics covered, within the general areas listed above, included:

- Moonshine and the Monster;
- McKay correspondence;
- Finite groups;
- Hyperbolic geometry;
- Galois theory;
- Modular groups and functions.

By all accounts of those who took part, the workshop was a very great success.

John Harnad
Director, Mathematical Physics Laboratory
Centre de recherches mathématiques, and
Department of Mathematics and Statistics
Concordia University

Montréal, December 2008

Organisateurs / *Organizers :*

John Harnad (Concordia, CRM)
François Lalonde (Montréal, CRM)
Pavel Winternitz (Montréal, CRM)

Sujets principaux / *Principal Topics :*

Moonshine et le monstre / *Moonshine and the Monster*
Correspondance de McKay / *McKay correspondence*
Groupes finis / *Finite groups*
Théorie de Galois / *Galois theory*
Groupes et fonctions modulaires / *Modular groups and functions*
Calcul algébrique / *Algebraic computation*

Participants invités / *Invited participants include :*

Hélène Airault (Picardie)
Dan Allcock (Austin)
Michael Aschbacher (Caltech)
Philip Boalch (ENS, Paris)
Abdllilah Bouali (Bordeaux)
* Pierre Cartier (IHES, Paris)
David Casperson (UNBC)
Harvey Cohn (CUNY)
Chris Cummins (Concordia)
Henri Darmon (McGill)
Jorge Andres Devoto (Buenos Aires)
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Hershy Kisilevsky (Concordia)
Dmitry Korotkin (Concordia)
Emmanuel Letellier (Concordia)
Kefeng Liu (UCLA)
Yuri Manin (Max Planck Institute, Bonn)
Geoffrey Mason (UC Santa Cruz)
John McKay (Concordia)
Jack Morava (Johns Hopkins)
Ram Murty (Queen's)
Iku Nakamura (Hokkaido)
Simon Norton (Cambridge)
Yousuke Ohyama (Osaka)
Dana Schlomiuk (Montréal)
Norbert Schlomiuk (Montréal)
Abdellah Sebbar (Ottawa)
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* Leon Takhtajan (SUNY, Stony Brook)
Lee-Peng Teo (Malaysia)
Allan Trojan (York)
Pavel Winternitz (Montréal, CRM)

(*) À confirmer / *To be confirmed*

Les actes de la rencontre, augmentés d'articles invités, seront publiés dans un volume de la collection CRM-AMS Proceedings and Lecture Notes diffusée par l'American Mathematical Society, voir :

The proceedings of the conference, together with some further invited papers, will be published as a special issue of the CRM-AMS Proceedings & Lecture Notes series published by the American Mathematical Society. See :

http://www.ams.org/cgi-bin/bookstore/bookpromo/crmpseries

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Renseignements et inscription / *Information & registration*

Pour plus de renseignements sur le CRM, la région de Montréal, le plan du campus et la façon d'y accéder, les transports en commun et des suggestions d'héberge-ment, voir :

For CRM contact information, links to Montreal area and campus maps, information about access, local transportation, and suggested accommodations see :

www.crm.math.ca/Mckay07

UNIVERSITÉ Concordia UNIVERSITY

CENTRE DE RECHERCHES MATHÉMATIQUES

Groupes et symétries : des Écossais du néolithique à John McKay

Groups and symmetries: From the Neolithic Scots to John McKay

April 27–29 avril 2007

Une rencontre sur le développement de la théorie des groupes et ses aspects computationnels, pour honorer quatre décennies de contributions de la part de John McKay (et quatre millénaires de la part des Écossais).

A conference on the development of group theory and its computational aspects, in honour of four decades of contributions by John McKay (and four millennia by the Scots).

La rencontre aura lieu au Centre de recherches mathématiques, sur le campus de l'Université de Montréal.

To be held at the Centre de recherches mathématiques, on the campus of the Université de Montréal

Parrainé par / Sponsored by : Concordia University & Centre de recherches mathématiques

"The Platonic solids", Scotland, ca. 2000 B.C.
(Photo: Graham Challifour, from "Time Stands Still", K. Critchlow.)

www.crm.math.ca/McKay07

Conception graphique / Graphic Design : Néograf Design 450-471-4980

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J. Conway and V. V. Nikulin

J. Harnad

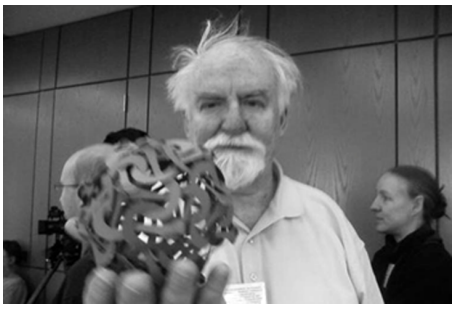
T. Gannon and C. Cummins

N. M. Katz

People Sculpture and J. Conway

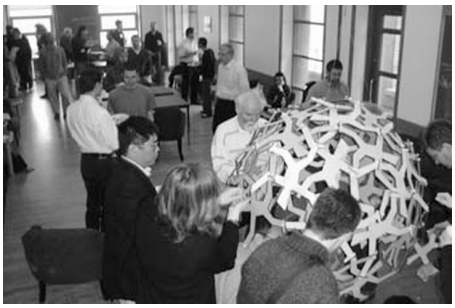
Creating People Sculpture

J. McKay



J. McKay

J. Hurtubise, E. Goren,
and S. P. Norton



Y. Ohyama and J. McKay

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This volume contains papers presented at a conference held in April 2007 at the CRM in Montreal honouring the remarkable contributions of John McKay over four decades of research. Papers by invitees who were unable to attend the conference are also included.

The papers cover a wide range of topics, including group theory, symmetries, modular functions, and geometry, with particular focus on two areas in which John McKay has made pioneering contributions: "Monstrous Moonshine" and the "McKay Correspondence". This book will be a valuable reference for graduate students and researchers interested in these and related areas and serve as a stimulus for new ideas.

ISBN 978-0-8218-4481-6

9 780821 844816

CRMP/47

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