

Volume 50

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PROCEEDINGS &  
LECTURE NOTES

Centre de Recherches Mathématiques  
Montréal

A Celebration of  
the Mathematical  
Legacy of Raoul Bott

P. Robert Kotiuga  
*Editor*



American Mathematical Society

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## A Celebration of the Mathematical Legacy of Raoul Bott

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The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FQRNT), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal. The CRM may be reached on the Web at [www.crm.math.ca](http://www.crm.math.ca).



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The compositions For Liv and Invention appear courtesy of the Bott family.

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## Preface

Raoul Bott was a “citizen of the world” in that he was born in Budapest Hungary in 1923, moved to Slovakia soon after, and when he was 16, spent a year being schooled in England. He then lived in Canada for six years, where he obtained two degrees in Electrical Engineering from McGill University, during which time he resolved to become a mathematician. Bott finished a ScD in applied mathematics under Richard Duffin at CMU (which at that time was Carnegie-Tech), and solved the most challenging synthesis problem in electrical network theory at that time. An encounter with Herman Weyl landed him an invitation to the IAS at Princeton where, after a few relatively intense incubation years, the application of Morse theory to the problem of computing geodesics on Lie groups, and the Bott periodicity theorem for the stable homotopy groups of the classical Lie groups, helped metamorphose him into one of the most influential topologists of the 20th century. In a period of about a decade, he was awarded tenure at the University of Michigan and then at Harvard, and among his graduate students, two (Daniel Quillen and Stephen Smale) would eventually be awarded Fields medals for their work in topology. After four decades at Harvard and many more students, he retired to San Diego where he succumbed to lung cancer late in 2005.

Raoul Bott’s collected papers testify to his contributions to mathematics. This book, like the conference that preceded it, strives to feature prominent researchers who see further by standing on Bott’s shoulders. By looking forward, neither project was a systematic attempt to summarize the mathematics found in the four volumes of his collected works, published over a decade ago. Rather, we are trying to develop a view of how the mathematics Bott mastered is manifested in current mathematical research and in emerging applications to mathematical physics.

P. Robert Kotiuga

# For Liv

Raoul Bott

Moderato

Musical score for "For Liv" by Raoul Bott, marked Moderato. The score is in 3/4 time and consists of five systems of piano accompaniment. The first system starts with a treble clef and a common time signature, which changes to 3/4. The second system starts at measure 8. The third system starts at measure 17. The fourth system starts at measure 24. The fifth system starts at measure 29 and includes triplet markings. The piece concludes with a double bar line.

# Invention

Raoul Bott

Moderato

The musical score is written for piano in G major (one sharp) and 3/4 time. It consists of four systems of music, each with a treble and bass clef staff. The tempo is marked 'Moderato'. The piece begins with a simple harmonic progression in the right hand and a steady bass line in the left hand. The first system (measures 1-5) establishes the key and rhythm. The second system (measures 6-10) introduces more complex rhythmic patterns in the right hand, including eighth and sixteenth notes. The third system (measures 11-15) continues with similar patterns, showing the development of the melody. The fourth system (measures 16-20) concludes the piece with a final cadence, marked with a double bar line and a repeat sign.



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A five-day conference celebrating the legacy of Raoul Bott was held at the CRM on June 9–13, 2008. The conference focused on the extraordinary impact Bott had on both topology and interactions between mathematics, physics and technology. The conference was co-organized by the Clay Mathematics Institute and had support from the National Science Foundation (Award 0805925). Montréal was a natural venue for such an event since Raoul Bott obtained two degrees in Electrical Engineering at McGill University in the 1940s and an Honorary Doctorate from McGill in 1987. The fact that Bott's presence is still fresh in the minds of all those involved made for a tremendous amount of enthusiasm and every attempt has been made to channel this energy into this book.

The contributions to this book come from three generations of Bott's students, coauthors, and fellow kindred spirits in order to cover six decades of Bott's research, identify his enduring mathematical legacy, and the consequences for emerging fields. The contributions can be read independently. In order to help a whole to emerge from the parts, the book is broken into four sections and to make the book accessible to a wide audience, each section starts with easier-to-read reminiscences and works its way into more involved papers.

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