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LECTURE NOTES

Centre de Recherches Mathématiques
Montréal

Spectrum and Dynamics

Proceedings of the Workshop
Held in Montréal, QC
April 7–11, 2008

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The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FQRNT), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal. The CRM may be reached on the Web at www.crm.math.ca.

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Introduction

The workshop on *Spectrum and Dynamics* was held in April 2008 at the Centre de Recherches Mathématiques in Montréal. The goal of this meeting was to bring together experts working in two different and yet interrelated areas of mathematics: geometric spectral theory and theory of dynamical systems.

The connection between these two fields goes back to the correspondence principle, formulated by Niels Bohr around 1920. It predicts that the behavior of a quantum system in the high energy (semiclassical) limit should reflect the properties of the corresponding classical dynamical system.

One of the most important manifestations of this principle is the quantum ergodicity property satisfied by the Laplace – Beltrami eigenfunctions on negatively curved manifolds: it states that the huge majority of high-energy eigenfunctions are "equidistributed" over the manifold. This property originates from a short paper by A. Shnirelman published in 1974. Since then, the study of "quantum limits" (i.e., the possible asymptotic shapes taken by high-energy eigenfunctions) has been rapidly developing, particularly due to links with number theory and mathematical physics. Some recent achievements in the field of "quantum chaos" are described in the lecture notes of S. Nonnenmacher, that are based on his work with N. Anantharaman and H. Koch. The central result of these notes is a lower bound on the Kolmogorov – Sinai entropy of quantum limits, which implies a "partial delocalization" of all high-energy eigenstates. Such a bound holds for a rather general class of quantum systems, obtained as quantizations of Hamiltonian flows (or diffeomorphisms) of Anosov type, Laplace eigenfunctions on negatively curved manifolds being an important special case. Recent developments related to the celebrated Quantum Unique Ergodicity conjecture of Rudnick – Sarnak are also reviewed in these notes.

The study of quantum limits for differential operators on vector bundles (such as the Hodge Laplacian, the Dirac operator or the Laplacian on forms) is the subject of the survey by A. Strohmaier. In particular, the analogue of Shnirelman's quantum ergodicity theorem for such operators is discussed. The underlying classical system in this setting is the frame flow. Its dynamical properties are quite different from those of the geodesic flow, which makes the topic particularly interesting. The survey is based on the recent joint works of the author with D. Jakobson and S. Zelditch.

An overview of results on classical and quantum dynamical systems associated with transverse Dirac operators on Riemannian foliations is presented in the contribution of Yu. Kordyukov. Well-known results such as Egorov's theorem are extended to the leaf space of a Riemannian foliation, using ideas and notions from noncommutative geometry. The author also considers two examples of Riemannian

foliations: a foliation given by the fibers of a fibration, and a linear foliation on the two-dimensional torus.

In his contribution, entitled *Eigenvalue variation and semiclassical concentration*, L. Hillairet considers a semiclassical Schrödinger operator on $\mathbb{R}^d$, and shows that a (semiclassical) limit point of an eigenbranch is necessarily a critical point of the potential. He uses a connection between the behavior of analytic eigenvalue branches, and concentration properties of the corresponding eigenfunctions. Similar ideas are applied to prove an "averaged" remainder estimate in Weyl's law on a smooth compact manifold, where the averaging is taken over an analytic variation of metrics.

The article by H. Lapointe is concerned with spectral asymptotics on tori with Liouville metrics. The geodesic flow on such tori is completely integrable. From the dynamical viewpoint, integrable metrics are "opposite" to the negatively curved metrics, whose geodesic flow is ergodic. Developing the techniques of Colin de Verdière and Bleher – Minasov – Kosygin – Sinai, the author reduces the eigenvalue counting problem for the Laplacian on a Liouville torus to a certain lattice counting problem. This allows him to obtain a van der Corput-type estimate on the error term in Weyl's law for a large class of Liouville tori.

The workshop featured a high level of diversity of topics, which is reflected in the current volume. Some of the talks were "purely spectral," some were "purely dynamical." This gave an opportunity for the participants to learn new mathematics, and, hopefully, to uncover yet unknown links between spectral theory and the theory of dynamical systems.

The article by H. Donnelly is concerned with the spectral properties of complete manifolds of negative curvature. The simplest example of such a manifold is the hyperbolic space of constant negative curvature that has purely continuous spectrum. It has been previously shown by several authors that the absence of embedded eigenvalues also holds for manifolds, whose curvature (as a function of geodesic distance from a given basepoint) converges from below to a negative constant rapidly enough. In the present paper the opposite phenomenon is exhibited: the author constructs examples of complete negatively curved manifolds with eigenvalues embedded in the continuous spectrum.

The paper by B. Helffer and T. Hoffmann-Ostenhof is concerned with the study of spectral minimal partitions. This is a new topic in geometric spectral theory, which is being developed by the authors and their collaborators. The problem is to find a k-partition of a given domain that minimizes the maximum among fundamental tones (i.e., the first Dirichlet eigenvalues) of the subdomains. For $k = 2$, a spectral minimal partition is always given by the nodal domains of a second eigenfunction. The case $k = 3$ is already highly nontrivial, and is considered in detail in the paper. In particular, the authors discuss an intriguing conjecture that the minimal 3-partition of a disk is given by the "Mercedes star."

In his paper, *Asymptotic vertex growth for graphs*, M. Pollicott studies discrete analogues of relatively old results about critical metrics for the entropy functional on Riemannian manifolds, proved by Katok and Besson, Courtois and Gallot. The author considers the entropy on weighted graphs, showing that it is analytic in the edge weights and convex, and determining the extremal weight assignment for the entropy on a given graph. He also studies analogous extremal problems for

asymptotic weighted vertex growth (a discrete analogue of the topological pressure) for a real-valued function on the vertices of the graph.

The symbolic coding of the geodesic flow on the Hecke surfaces $G_q\backslash\mathbb{H}$ (where G_q is a Hecke triangle group), is related to nearest λ_q-multiple continued fractions for $\lambda_q = 2\cos(\pi/q)$, discussed in the work by D. Mayer and T. Mühlenbruch. These fractions are generated by interval maps f_q which are conjugate to subshifts over infinite alphabets. They have been introduced for $q = 3$ by Hurwitz and for even q by Nakada. The authors generalize to arbitrary q a result of Hurwitz concerning the G_q- and f_q-equivalence of points on the real line. They prove that the natural extension of f_q can be used as a Poincaré map for the geodesic flow on $G_q\backslash\mathbb{H}$, and allows one to construct the transfer operator for this flow.

In his paper, entitled *Comparing length functions on free groups*, R. Sharp studies pairs of length functions on a free group, associated with pairs of points in the outer space. The author discusses several invariants of these pairs, and describes the connection to an associated "Manhattan curve" analogous to those considered for pairs of hyperbolic surfaces.

The workshop on *Spectrum and Dynamics* was yet another illustration of the unity of mathematics. Analysis, geometry, dynamics, number theory, group theory, mathematical physics — it is hard to find a branch of mathematics that was not present in the talks and discussions!

Many important developments and new ideas were reported at the meeting. For the organizers, it was an interesting and gratifying experience. However, all of this would have been impossible without the daily assistance from the dedicated staff of the Centre de Recherches Mathématiques, in particular, Louis Pelletier and Josée Laferrière. Their indispensable help in the organization of the workshop is greatly appreciated.

Dmitry Jakobson
Stéphane Nonnenmacher
Iosif Polterovich

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This volume contains a collection of papers presented at the workshop on Spectrum and Dynamics held at the CRM in April 2008. In recent years, many new exciting connections have been established between the spectral theory of elliptic operators and the theory of dynamical systems. A number of articles in the proceedings highlight these discoveries. The volume features a diversity of topics, such as quantum chaos, spectral geometry, semiclassical analysis, number theory and ergodic theory. Apart from the research papers aimed at the experts, this book includes several survey articles accessible to a broad mathematical audience.

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