

Volume 53

CRM

# CRM PROCEEDINGS & LECTURE NOTES

Centre de Recherches Mathématiques  
Montréal

## Models, Logics, and Higher-Dimensional Categories

A Tribute to the Work of  
Mihály Makkai

Bradd Hart  
Thomas G. Kucera  
Anand Pillay  
Philip J. Scott  
Robert A. G. Seely  
*Editors*



American Mathematical Society

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## Introduction

On June 18th–20th, 2009, a meeting was held at the Centre de recherches mathématiques (at l’Université de Montréal) as a tribute to the work of Mihály Makkai. This was supplemented by a one-day workshop on July 18th at McGill University. Such a meeting was timely for several reasons: not only the occasion of Makkai’s 70th birthday year, but also because of the importance and influence his work continues to hold in a wide range of areas. For example, there is his work in infinitary logic, admissible sets and traditional model theory; his papers in the early 80’s introduced a whole generation of model theorists to stability theory. The 1960s and 70s saw the development of a considerable body of work creating a remarkably successful marriage of the ideas and techniques of category theory on the one hand and logic and model theory on the other hand. Makkai was and has continued to be a leading and original contributor to this endeavour. The monograph *First Order Categorical Logic* (Makkai & Reyes, 1977) was the first to successfully combine traditional model theory and (infinitary) logic with the work of the Grothendieck school in algebraic geometry; it was a milestone in the subject and set the stage for subsequent work. In succeeding years, Makkai has built up an impressive body of work in several related fields: categorical model theory, first order logic with dependent sorts, higher dimensional category theory, and most importantly, a coherent and far reaching view of categorical logic in mathematical foundations. We look forward to a greater integration of this categorical point of view into contemporary model theory, some examples of which are already present in this volume.

The meeting focussed on the main themes in Makkai’s research career: traditional model theory, categorical model theory and logics, and higher-dimensional category theory. This volume is a partial proceedings of this event, and displays the vibrant nature of current research reflecting the influence of Makkai’s ideas and his work. One notable sentiment often expressed at the meeting was the affection and regard all the participants felt for Mihály. Most of us are long-time colleagues, many are former students or postdocs of his (a blend reflected in the editorship of this volume), and have long benefited from his advice and example. It is with great pleasure we offer this tribute of our respect and friendship to him.

The Editors



## Mihály Makkai: A Biographical Note, with Reminiscences

Mihály Makkai was born in Budapest on June 24, 1939, the son of László Makkai, a noted Hungarian historian, and the grandson of Sándor Makkai, a well-known writer and a bishop of the Hungarian Reformed Church in Transylvania. At the conference, former student Djordje Čubrić treated us to a brief account of Mihály's very early career as a mathematician: not the first professional publications from 1964 and 1965, but rather his frequent appearances in the early 1950s as a problem-solver in the famous Hungarian high-school mathematics magazine *KöMaL*.

Mihály was a student of Róza Péter at the Eötvös Loránd University in Budapest, and was profoundly influenced by studies with Andrzej Mostowski in Warsaw in 1964–1965. He obtained his Ph.D. in 1966 and was awarded the Candidate's Degree in Mathematics from the Hungarian Academy of Sciences in 1969.

A significant early event for Mihály was a year (1966–1967) spent as a guest lecturer at University of California, Berkeley, where he taught two terms of model theory: Ehrenfeucht–Mostowski models, saturation and homogeneity, and Morley's Categoricity Theorem in the first term, and infinitary logic in the second. Julia Knight spoke at the conference of the impact that this course had on her and many other logic students; this was also the beginning of a long friendship. Already one of Mihály's best habits as a teacher was apparent: he gave many long written assignments and returned them to his students covered with detailed comments not only on errors, but on other ideas suggested by the solutions, or on alternate solutions. This is a practice still followed by many of his students in their teaching of logic, including the author of this note. As well, it was here that Mihály first met his long-time friend and collaborator Gonzalo Reyes.

In 1971, Mihály and his family were able to leave Hungary, and at the invitation of another well-known Hungarian mathematician George Grätzer, joined the Department of Mathematics and Astronomy at the University of Manitoba. It was here that the author of this note first encountered him: I was just at the end of the second year of my undergraduate program, when the Head of the Department tracked me down and told me that “We have to find some students for this new fellow from Hungary; you will be taking his fourth-year Honours logic course next year.” I was thoroughly intimidated at first, but it was only four or five weeks into the course the following autumn I knew what I was going to be doing for the rest of my life. Mihály soon moved to Montréal, and after a year as a Research Associate at the Centre de recherches mathématiques, joined the faculty at McGill University, where he has been ever since. He was appointed Peter Redpath Chair of

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Compiled from information provided by Mihály Makkai, Djordje Čubrić, Victor Harnik, Julia Knight, and from the personal recollections of these, the author, and others.

Pure Mathematics in 2000, and received the David Thompson Award for Graduate Teaching and Supervision in 2002.

At the conference, many others also spoke of the electric effect of Mihály's teaching on their mathematical careers. Hal Kierstead, while a student of Alfred Manaster's at University of California, San Diego, first met Mihály as a visiting student at the special year in Mathematical Logic at UCLA during 1975–1976. Hal followed Mihály to Montréal and McGill University, and (under the joint supervision of Manaster and Makkai) was the first student to receive a Ph.D. under Mihály's direction. Mihály's teaching style at all times reveals his passion for the subject matter, and whether it is in the classroom, in discussions with students, or at work with a collaborator, his obvious joy at the beauty of mathematics is contagious.

Mihály's published works (well over 80 as of this writing) show a very wide range of interests in foundational matters. His early reputation was founded on strong contributions to infinitary logic, and this interest has remained throughout his career. His reconnection with Reyes at the Centre de recherches mathématiques in 1973–1974 began the long series of papers in categorical logic, topos theory, and, more recently, higher dimensional category theory. At the same time he became deeply interested in Shelah's Classification Theory, with his 1984 paper *A survey of basic stability theory* serving as a guide for young researchers for years. Several people at the conference commented on the influence of this paper. It is a prime exemplar of Mihály's mathematical writing style: clear, well-organized, and direct, and always with the "right" way of motivating and explaining difficult ideas. The foundations of the work in categorical logic were laid with the Springer Lecture notes book with Reyes, *First order categorical logic*, followed 11 years later with the AMS Monograph *Accessible categories*, with R. Paré. Mihály's intuition about the importance of analogues of Stone Duality in first order logic were expressed in several papers, culminating in *Strong conceptual completeness for first order logic*; a paper presenting an entirely new kind of completeness theorem for first-order logic which could only be expressed using the language and tools of categorical logic. This body of work makes significant contributions towards understanding what it means to "interpret one theory in another." In recent years, Mihály (with students and collaborators) has studied higher-dimensional categories. This volume contains work by some of his collaborators in this domain.

Throughout his career, in research and in teaching, Mihály's work has been motivated by a desire to understand the foundations of mathematics in the broadest sense. This volume amply illustrates this theme.

Thomas G. Kucera

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The book deals with the main themes in Mihály Makkai's research career: traditional model theory, categorical model theory and logics, and higher-dimensional category theory. Included are both research papers and survey papers, giving useful material for experts and students in these fields. Particularly valuable are papers that show how the techniques and understanding in one field can be productively applied to another; examples are the paper by Harnik, which explains how Shelah's  $T^{\text{eq}}$  construction (in model theory) is the same as the categorical notion of pretopos completion; the paper by Kamensky, which gives category-theoretic treatments of sophisticated notions from stability theory; and the paper by Prest relating categorical logic and the model theory of modules. These and other papers in the volume should make this a valuable resource for any mathematician interested in classical or categorical model theory, or higher dimensional category theory.

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