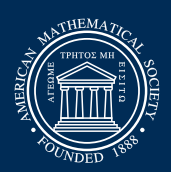
Volume 55

CRM PROCEEDINGS & LECTURE NOTES

Centre de Recherches Mathématiques
Montréal

Complex Analysis and Potential Theory

André Boivin
Javad Mashreghi
Editors



American Mathematical Society

Complex Analysis and Potential Theory

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The Centre de Recherches Mathématiques (CRM) of the Université de Montréal was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM is supported by the Université de Montréal, the Province of Québec (FQRNT), and the Natural Sciences and Engineering Research Council of Canada. It is affiliated with the Institut des Sciences Mathématiques (ISM) of Montréal. The CRM may be reached on the Web at www.crm.math.ca.

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Photo of Paul Gauthier courtesy of M. Peter Krasselt. Photo of Kohur N. GowriSankaran courtesy of Kohur N. GowriSankaran.

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10 9 8 7 6 5 4 3 2 1 17 16 15 14 13 12

Contents

Preface vii

A Self-Contained Proof of the Strong-Type Capacitary Inequality for the Dirichlet Space
Omar El-Fallah, Karim Kellay, Javad Mashreghi, and *Thomas Ransford* 1

A Simple Numerical Approach to the Riemann Hypothesis
N. Tarkhanov 21

A Survey of Linear Extremal Problems in Analytic Function Spaces
Catherine Bénéteau and *Dmitry Khavinson* 33

A Unifying Construction for Measure-Valued Continuous and Discrete Branching Processes
Lucian Beznea, Oana Lupaşcu, and *Andrei-George Oprina* 47

Compactifications of the Plane and Extensions of the Disc Algebra
V. Nestoridis 61

Examples of Quantitative Universal Approximation
Thomas Kalmes, Markus Nieß, and *Thomas Ransford* 77

Harmonic Mappings with Quadrilateral Image
Jane McDougall 99

Hartogs Phenomenon on Unbounded Domains—Conjectures and Examples
Al Boggess, Roman J. Dwilewicz, and *Zbigniew Słodkowski* 117

Integration Formulae and Kernels in Singular Subvarieties of $\mathbb{C}^n$
Luis M. Hernández-Pérez and *Eduardo S. Zeron* 135

Invariant Potential Theory, Derivatives of Inner Functions, and $B^{p,q}$ Spaces in the Unit Ball of $\mathbb{C}^n$
Manfred Stoll 149

Logarithmic Hölder Estimates of p-Harmonic Extension Operators in a Metric Measure Space
Tsubasa Itoh 163

Meromorphic Approximation on Noncompact Riemann Surfaces
Nadya Askaripour and *André Boivin* 171

On a Family of Outer Functions
Javad Mashreghi 193

On C^m-Subharmonic Extension Sets of Walsh-Type
P. V. Paramonov 201

On Maximal Plurisubharmonic Functions
A. Sadullaev 211

On Universality of Series in Banach Spaces
Richard Fournier and *Jérôme-Melville Giguêre* 217

Orlicz Capacity of Balls
Yoshihiro Mizuta and *Takao Ohno* 225

Potential Analysis on Nonsmooth Domains—Martin Boundary and Boundary Harnack Principle
Hiroaki Aikawa 235

Potential Theory on Trees and Multiplication Operators
David Singman 255

Recent Progress on Fine Differentiability and Fine Harmonicity
Stephen J. Gardiner 283

Reversibility Questions in Groups Arising in Analysis
Anthony G. O'Farrell 293

Subordinate Harmonic Structures in an Infinite Network
Victor Anandam 301

The Generalized Binomial Theorem
Murali Rao 315

Uniform and C^m-Approximation by Polyanalytic Polynomials
Konstantin Yu. Fedorovskiy 323

Preface

A four-day international conference entitled "Complex Analysis and Potential Theory" was held in Montréal on June 20–23, 2011 at the Centre de recherches mathématiques (CRM). More than 50 mathematicians from 15 countries participated in the conference. The program consisted of 24 invited presentations and 14 contributed talks. The lectures given by some of the most established specialists in the fields covered a large range of topics: Geometric Function Theory; Approximation Theory; Operator Theory; Function Spaces; Classical, Abstract and Probabilistic Potential Theory; Pluripotential Theory; Harmonic Analysis; Orthogonal Polynomials; Complex Dynamics; and many others. The conference has provided means to bring together specialists, young researchers and graduate students from both Complex Analysis and Potential Theory to foster further cooperation and exchange of ideas and techniques to find new research perspectives.

The Conference was held to honour the important contributions to mathematics and the mathematical community of two influential analysts from Montréal: Professor Kohur N. GowriSankaran (McGill University) and Professor Paul M. Gauthier (Université de Montréal), both of whom have had long and distinguished careers in Montréal extending over four decades. Short biographies are provided below.

This proceedings contains 24 surveys and research articles from the participants and their co-authors. These papers are dedicated to Professors GowriSankaran and Gauthier. From potential theory on trees to approximation on Riemann surfaces, from universality to inner and outer functions and the disc algebra, from branching processes to harmonic extension and capacities, from harmonic mappings and the Harnack principle to integration formulae in several complex variables and the Hartogs phenomenon, from fine harmonicity and plurisubharmonic functions to the binomial identity and the Riemann hypothesis, and more, the diversity of subjects reflects the wide range of research interests of the two honored guests at this conference.

André Boivin
Javad Mashreghi

Paul M. Gauthier

Kohur N. GowriSankaran

Paul M. Gauthier

Paul is Franco-American. That is, his parents were French Canadians and he was born in the U.S. (in 1940). In 1967, Paul completed his military service in the National Guard and his doctoral studies at Wayne State University, under the direction of Wladimir Seidel. Seidel was a student of Constantin Carathéodory and was one of the founders of the theory of cluster sets and also of inner functions.

In 1967, Paul returned to the land of his ancestors, Canada, to join Université de Montréal. He was quite active in the Canadian Mathematical Society, where he served as Vice-president from 1993 to 1995. He was twice director of CMS summer research institutes and was scientific director for a meeting of the CMS. For many years, he was on the human rights committees of the CMS as well as the AMS and in both he was chair of the committees at some time. He was also scientific director for a Séminaire de mathématiques supérieures at Université de Montréal.

Paul has visited numerous institutions in his career. In particular, this list includes University of Maryland, 1972–1973; Wayne State University, May 1973; Universität Konstanz, fall 1974; Université de Paris, spring 1975; Steklov Institute, 1981–1982, 1989–1990; Bar Ilan University, May 1987; Beijing University, 1996–1997, fall 2003; Universität Oldenburg, summer 2003; Universität Potsdam, November 2003, summer 2007, spring 2010; Centre de Recerca Matemàtica, Barcelona, winter–spring, 2007; Jammu University, September 2010. He lectured also in Armenia, Austria, Bulgaria, China, Cyprus, Denmark, Finland, France, Gaza, Germany, Georgia, Greece, Hong Kong, Iran, India, Ireland, Israel, Italy, Japan, South Korea, Malaysia, Mexico, Morocco, Poland, Romania, Russia, Saudi Arabia, Soviet Union, Spain, Switzerland, Taiwan, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, United States, Uzbekistan, Vietnam and Yugoslavia.

Paul has given plenary lectures at the Canadian Mathematical Society meeting in 1983, twice at the Colloque des mathématiques du Québec, at the Irish Mathematical Society meeting in 1994, at the Saudi Arabia Mathematical Society meeting in 2002, and in the Iranian Mathematical Society meeting in 2005.

Paul has more than 130 publications (recorded in Zentralblatt). In particular, he has translated four books, and several important articles, from Russian to English. At the Ph.D. level, Paul has had 11 students who completed and 17 descendants. He directed many students at the master's level and several postdoctoral students. In recent years, he has also directed four undergraduate research students, all of whom have published the results of their work.

Since retirement in 2008, Paul has been adjunct professor at Université de Montréal and membre régulier of the Centre de recherches mathématiques. At the time of this writing he has one postdoctoral student. In the past, Paul's presentations have been in complex analysis and potential theory, but in 2012, Paul also gave a talk at a meeting on algebraic geometry and an invited talk on the Riemann hypothesis, in a special session of the AMS on number theory.

Kohur N. GowriSankaran

Already in his twenties while at the Tata Institute of Fundamental Research (TIFR) Bombay he demonstrated his skill and vision. He was chosen to occupy for about a year before he left for the States in 1967, the position of the Secretary of the powerful Library Committee of the TIFR. This Committee was responsible for policy decisions of acquisition, hiring for Library, etc and had four other members who were all professor at the Institute. Though he was a junior member, he brought in policies for Library acquisition, particularly journals, in the face of tremendous foreign exchange problems that India faced at that time. He laid the groundwork for the future.

Soon after his arrival at McGill University in 1968, he and a few other colleagues were mainly responsible in establishing standards that are consistent with those practiced at some of the best universities, for the Preliminary Examinations for doctoral students.

During the next several years, he held positions in several Departmental and University committees. However, under the Chairmanship of Peter Russell, he agreed to take up the responsibility to chair the Committee on Graduate Affairs and become the Graduate Program Director in 1988, a position he held for four years. It is under his leadership that the graduate registration that had sunk to a low of about 30 in the fall of 1988 was brought up to over hundred in just a couple of years.

Once again, this time under the chairmanship of Georg Schmidt, he took up the position of Undergraduate Program Director and did it for three years during the period 1994-1998. In this role he established several important policies including cooperation with other departments which is very critical for Mathematics departments.

It is in 1994 he realized that there are in general very few mathematicians who participate at policy making and governance at the senior level of the University. He decided to make his presence felt at that level. He started off by getting elected to Senate of McGill University and got elected to the powerful Senate Steering Committee (1999–2010). He served four continuous elected terms on the Board of Governors and during some of those years was on the Executive Committee of the Board. He chaired the University Committee on Scholarships and Student Aid from 1999–2002. He was also chosen to become the President of the McGill Association of University Teachers (MAUT) for a term and he was on the Executive Committee from 2001–2004. The indirect benefit of increasing the profile of Mathematics from these activities is enormous. There are very few mathematicians who take up this kind of challenge unlike for example chemists, exceptions in the recent years in Canada (to the best of our knowledge) being Luc Vinet (Recteur at Université de Montréal) and Eddy Campbell (Provost at Memorial University and now President at The University of New Brunswick).

He was invited by the Dean of the Faculty of Science in 1999 to chair the Department. Despite being heavily involved in University Committees, he accepted the challenge and chaired the Department for six years until June 1, 2005. Among his many accomplishments, he made altogether almost 20 new faculty appointments during those six years. He brought enormous talent, both at the junior and senior level, to set the standard and ensure a very bright future for the Department for the coming twenty to thirty years.

He has been a member of the Canadian Mathematical Society for many years. He was named to a couple of committees in the eighties. He participated very vigorously as a chair during the last six years. In particular, he worked hard to make sure the meeting of the CMS held in Montréal in December of 2004 was a huge success. In the process he also raised several thousand dollars for CMS in conjunction with the meeting held in Montréal.

All along he never for any length of time forgot the raison d'être: Mathematics and research. He continues to be very active in research and publishes in some of the leading journals (Potential Analysis, American Journal of Mathematics, etc.). He has been invited to a large number of subject related conferences in several countries like France, Germany, Czech Republic, Romania, Tunisia, etc. and has given over six plenary talks just in the past decade. A total of nine students successfully completed their Ph.D. degrees under his supervision. That is one of the highest numbers in the entire history of the department, matched by a couple of other colleagues. Besides those his contribution to research has been in another dimension too. He was successful in getting funding directly from NATO (not an institutional process) to hold an Advanced Research Workshop in France (at Château de Bonas) in 1993. This was supplemented by an additional grant by the French Government! He has organized many special sessions at meetings of the American Mathematical Society as well as one under the banner of ACFAS (Association francophone pour le savoir).

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