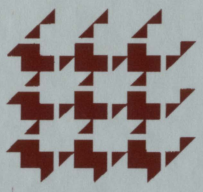


DIMACS

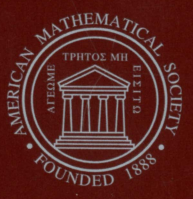
Series in Discrete Mathematics and Theoretical Computer Science

Volume 39

Proof Complexity and Feasible Arithmetics

DIMACS Workshop
April 21–24, 1996

Paul W. Beame
Samuel R. Buss
Editors



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Proof Complexity and Feasible Arithmetics

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NSF Science and Technology Center
in Discrete Mathematics and Theoretical Computer Science
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This DIMACS volume contains papers from the DIMACS Workshop on Feasible Arithmetics and Length of Proofs, which was part of the DIMACS Special Year on Logic and Algorithms. The Workshop was held on April 21–24, 1996.

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Foreword

The DIMACS Workshop on "Feasible Arithmetics and Length of Proofs" held in April 1996 was part of DIMACS Special Year on Logic and Algorithms. We would like to express our appreciation to Paul Beame and Sam Buss for their efforts to organize and plan this successful workshop as well as editing this volume of papers.

The workshop was part of the broader Special Year on Logic and Algorithms program which focused on computer aided verification, finite models, and proof complexity. The special year encouraged collaborations among very different research communities and this volume records one of many workshops in which this was achieved. We also extend our thanks to Eric Allender, Robert Kurshan, and Moshe Vardi for their work over many months as special year organizers.

DIMACS gratefully acknowledges the generous support that makes these programs possible. The National Science Foundation, through its Science and Technology Center program, the New Jersey Commission on Science and Technology, DIMACS partners at Rutgers, Princeton, AT&T Labs, Bell Labs, and Bellcore generously supported the special year.

Fred S. Roberts
Director

Bernard Chazelle
Co-Director for Princeton

Stephen R. Mahaney
Associate Director for Research

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Preface

The complexity of proofs in propositional logic and the properties of feasible theories of arithmetic are closely related and give important and strong connections between logical properties and the properties of computational complexity classes.

Defining the proof complexity of logical formulas requires the specification of a particular *proof system* that determines the expressive power of and the relationships between the objects that may be used in their proofs. In addition, a proof system includes an efficient mechanism for checking the validity of proofs. Within such a proof system, the proof complexity of a formula is typically measured as the amount of space required to write down a proof. The study of proof complexity seeks to understand the complexity of proofs of formulas in specific proof systems, classify the relative complexity of proofs of formulas within different proof systems, and to develop better proof systems. For example, a major open question in proof complexity, equivalent to the NP versus co-NP problem, is whether or not there is a proof system in which all propositional tautologies have polynomial-size proofs.

Feasible theories of arithmetic are first-order theories of arithmetic designed to have proof-theoretic strength closely corresponding to low-level computational classes from the polynomial-time or NC hierarchies. The most natural feasible theories of arithmetic have the property that the functions which are provably total in the theory are precisely the functions which are computable in some given natural complexity class. Furthermore, it is frequently the case that first-order proofs in feasible theories of arithmetic can be translated directly into propositional proofs. The primary goals of the study of feasible theories of arithmetic are to understand the computational implications of various proof-theoretic constructions and use these to characterize complexity classes, and to establish connections between the logical properties of feasible theories and open problems in computational complexity. It is hoped that the study of feasible theories will ultimately yield useful new complexity results.

Over the last dozen years there has been substantial progress in proof complexity and feasible arithmetic as the two have grown together and have also benefited from a very productive cross-fertilization of techniques with Boolean circuit complexity. This progress has significantly broadened and deepened many connections of proof complexity and feasible arithmetic with computational complexity and greatly enriched both areas.

The papers in this volume represent the proceedings of a workshop on "Feasible Arithmetic and Proof Complexity" held at the DIMACS Institute at Rutgers, New Jersey on April 21-24, 1996. The principal speakers at this workshop included J. Avigad, P. Beame, S. Bellantoni, S. Buss, J.-Y. Cai, A. Carbone, P. Clote, F. Ferreira, X. Fu, R. Impagliazzo, J. Johannsen, B. Kauffmann, J. Krajíček, D. Leivant,

A.A. Razborov, K. Regan, S. Riis, T. Pitassi, P. Pudlak, S. Rudich, G. Takeuti and D. Willard, and many of these speakers submitted talks to this volume. The papers primarily cover lower bounds on the complexity of propositional proofs and meta-mathematical results on feasible theories of arithmetics. All papers in this volume are refereed.

We wish to thank the DIMACS institute for their generous support in hosting and financing the workshop and the DIMACS staff for their help in organizing the meeting. We also thank the referees for the rigorous and thorough review of the articles.

Paul Beame and Sam Buss

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