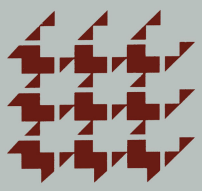


DIMACS

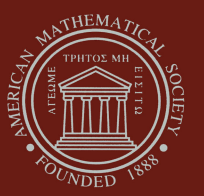
Series in Discrete Mathematics and Theoretical Computer Science

Volume 63

Graphs, Morphisms and Statistical Physics

DIMACS Workshop
Graphs, Morphisms and Statistical Physics
March 19–21, 2001
DIMACS Center

J. Nešetřil
P. Winkler
Editors



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Graphs, Morphisms and Statistical Physics

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Center for Discrete Mathematics
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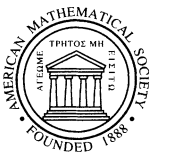
(with partners at Avaya Labs, IBM Research, and Microsoft Research)



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This DIMACS volume presents the proceedings from the workshop Graphs, Morphisms and Statistical Physics held at the DIMACS Center, Rutgers University, March 19–21, 2001.

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Foreword

A workshop on Graphs, Morphisms, and Statistical Physics was held at Rutgers University on March 19-21, 2001. We would like to express our appreciation to Jaroslav Nešetřil and Peter Winkler for their efforts to organize and plan this successful conference.

The workshop was part of a unique partnership between the DIMACS Center, headquarted in New Jersey, USA, and the DIMATIA Center, headquartered in Prague, Czech Republic. The partnership has included a series of workshops, an exchange of visitors, a joint undergraduate research (REU) program, and joint research projects.

Issues at the confluence of graph theory, computer science, probability, and statistical physics have come together in recent years to create exciting new research agendas. Areas of common interest include connections between random graphs and percolation, between slow mixing and phase transition, and between graph morphisms and hard-constraint models.

This volume is not simply a workshop proceedings. It includes papers from researchers whose work added to the coverage of the volume and also includes descriptions of work that was stimulated by the workshop particularly and the partnership between DIMACS and DIMATIA generally.

DIMACS gratefully acknowledges the generous support that makes these programs possible. In particular, the National Science Foundation supported this workshop, matched by a similar grant from its sister agency in the Czech Republic. In addition, the New Jersey Commission on Science and Technology, DIMACS' partners at Rutgers, Princeton, AT&T Labs-Research, Bell Labs, NEC Laboratories America, and Telcordia Technologies, and its affiliated partners at Avaya Labs, HP Labs, IBM Research, and Microsoft Research have generously supported the partnership between DIMACS and DIMATIA.

We are particularly grateful to DIMATIA and also to the Institute of Theoretical Computer Science (ITI) at Charles University for their support of our many joint projects and in particular for contributing so vitally to the success of the workshop and this volume.

Fred S. Roberts
Director

Robert Tarjan
Co-Director for Princeton

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Preface

The intersection of combinatorics and statistical physics has been an area of great activity over the past few years, fertilized by an exchange not only of techniques but of objectives. Spurred by computing theorists interested in approximation algorithms, statistical physicists and discrete mathematicians have overcome language problems and found a wealth of common ground in probabilistic combinatorics.

Close connections between percolation and random graphs, between graph morphisms and hard-constraint models, and between slow mixing and phase transition, have led to new results and new perspectives. These connections can help in understanding typical, as opposed to extremal, behavior of combinatorial phenomena such as graph coloring and homomorphisms.

Any "nearest neighbor" system of statistical physics can be interpreted as a space of graph morphisms—for example, morphisms to the two-node graph with one edge and one loop correspond to the "hard-core lattice gas model" and to random independent sets. Given the set of morphisms from a (possibly infinite) graph G to a graph H, when do we see long range order? When does changing the morphism site by site (heat bath) yield rapid mixing, or even eventual mixing? The special case of proper colorings (corresponding to the anti-ferromagnetic Potts model at 0 temperature) is especially interesting to graph theorists but more general notions of graph colorings may also yield some intriguing new questions.

Inspired by these issues, and encouraged by Fred Roberts and other colleagues, we organized a "DIMACS/DIMATIA Workshop on Graphs, Morphisms and Statistical Physics" which took place at Rutgers University, March 19–21, 2001. The workshop was attended by 53 scientists from the U.S. and abroad, and the present volume is an outgrowth of this meeting.

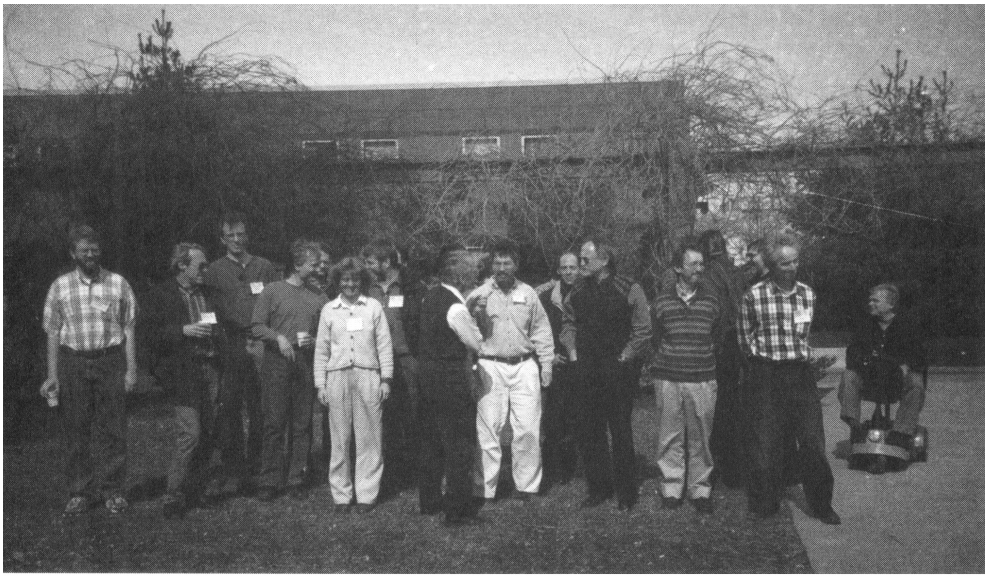
Some of the topics we cover here are: percolation, random colorings, homomorphisms from and to a fixed graph, mixing, combinatorial phase transitions, threshold phenomena, scaling windows, and some purely combinatorial aspects of graph coloring.

In addition to the participants we asked several other colleagues whose research complemented the scope of the volume. The volume also contains several contributions which were directly influenced by the fruitful atmosphere of this meeting. We thank all participants for their time and effort which made the success of this meeting possible.

This workshop was a joint activity of DIMACS, New Jersey and DIMATIA, Prague and it was supported by both of these centers. Most of the organization of the volume was done by Robert Šámal (DIMATIA–ITI) and without his effort the volume would hardly be possible.

The final stage of preparation of the volume was supported by Institute of Theoretical Computer Science (ITI) at Charles University, Prague (under grant LN00A056).

Jaroslav Nešetřil
Peter Winkler



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Inspired by issues and intriguing new questions surrounding the interplay of combinatorics and statistical physics, a DIMACS/DIMATIA workshop was held at Rutgers University. These proceedings are the outgrowth of that meeting. This volume is intended for graduate students and research mathematicians interested in probabilistic graph theory and its applications.

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