



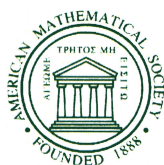
# FIELDS INSTITUTE COMMUNICATIONS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

## Nonlinear Dynamics and Time Series

Building a Bridge between  
the Natural and Statistical Sciences

Colleen D. Cutler  
Daniel T. Kaplan  
Editors



**American Mathematical Society**

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Providence, Rhode Island

## The Fields Institute for Research in Mathematical Sciences

The Fields Institute is named in honour of the Canadian mathematician John Charles Fields (1863–1932). Fields was a visionary who received many honours for his scientific work, including election to the Royal Society of Canada in 1909 and to the Royal Society of London in 1913. Among other accomplishments in the service of the international mathematics community, Fields was responsible for establishing the world's most prestigious prize for mathematics research—the Fields Medal.

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## Preface

The papers collected in this volume reflect an attempt to bring together researchers in two fields: Nonlinear Dynamics, and Statistics, with the goal of providing an exchange and creating a link between two diverse groups with a common interest in the analysis of nonlinear time series data.

The workshop in which these papers originated was entitled “Nonlinear Dynamics and Time Series: Building a Bridge Between the Natural and Statistical Sciences,” and was held at the Centre de Recherches Mathématiques (CRM) in Montréal, Canada in July 1995. The workshop was a joint venture funded partly by the CRM and partly by The Fields Institute for Research in the Mathematical Sciences. It followed in the footsteps of recent similar interdisciplinary meetings, such as the one-day meeting on chaos organized by the Royal Statistical Society in 1991, and the two-day meeting on chaos and forecasting sponsored by the Royal Society of London in 1994.

With ideas from nonlinear dynamics being applied more and more to real-world problems, it has become of increasing importance to develop and understand statistical methodology designed specifically for nonlinear and dynamical approaches to time series. And as nonlinear dynamicists attempt to cope with the limitations of noisy real-world data, the strategies and expertise of statisticians become of growing relevance and significance to them.

For statisticians, nonlinear dynamics offers new perspectives on the modelling and analyses of time series data, and has provided an influx of unfamiliar terminology, such as chaos, sensitivity to initial conditions, attractors, fractal dimension, embedding dimension,  $1/f$  noise, Lyapunov exponents, and bifurcations. Ideas from nonlinear dynamics stand to make an impact on the way in which statisticians view data, while at the same time presenting a wealth of open statistical problems to which statisticians can contribute their expertise, thereby providing a substantial return to the dynamics community.

Dynamicists and statisticians often need access to the concepts and tools developed in each discipline, but it can be difficult to cross the terminological and conceptual gulf that separates the two fields. We felt that by bringing together workers in both fields, charged with the task of making their own work accessible to those from the other side, we could lay the foundations for a bridge. For those who participated in the workshop, we feel that this ambition was fulfilled.



The normal mode of scientific communication, that of publication in peer-reviewed journals, is not always well suited to maintaining communication between fields. It can be daunting to pick up a journal from another field; the vocabulary and methods are often unfamiliar, and the motivation behind a particular approach (or even the significance and meaning of the problem being investigated) may be mysterious and obscure to the uninitiated. This problem is exacerbated by the fact that introductory material, essential to outsiders, is often suppressed in the name of conciseness and the desire to emphasize new results. It is even more difficult to write a paper for a journal in a different field; the reviewers may not be sympathetic to or cognizant of the methods and motivation behind the paper, or the style may simply jar them.

In reviewing the papers presented here, the editors and the anonymous peer-reviewers have attempted to minimize these problems. The result, we believe, is a collection of interrelated papers that highlight current areas of research in statistics that might have particular applicability to nonlinear dynamics, and new methodology and open data analysis problems in nonlinear dynamics that might find their way into the toolkits and research interests of statisticians.

We are of course very grateful for the funding, from both The Fields Institute and CRM, that made this workshop and volume possible. We thank John Chadam, then director of The Fields Institute, for providing the impetus to organize the workshop, and also the support staff, particularly Judy Motts of Fields and Louis Pelletier of CRM, for their excellent organizational work. A special note of thanks must go to the publications assistant Erna Unrau of The Fields Institute, who was responsible for getting this volume into the final form you see before you now, and who offered much sage advice throughout the publication process, not the least of which was “Relax, calm down, have a doughnut.”

Finally, and most importantly, we thank the participants and authors themselves, for contributing their time, talents, and enthusiasm to making the workshop a great success.

Colleen Cutler  
Danny Kaplan

August 1996

This book is a collection of research and expository papers reflecting the interfacing of two fields: nonlinear dynamics (in the physiological and biological sciences) and statistics. It presents the proceedings of a four-day workshop entitled "Nonlinear Dynamics and Time Series: Building a Bridge Between the Natural and Statistical Sciences" held at the Centre de Recherches Mathématiques (CRM) in Montréal in July 1995. The goal of the workshop was to provide an exchange forum and to create a link between two diverse groups with a common interest in the analysis of nonlinear time series data.

*An important interdisciplinary work ... provides a valuable collection of recent research ... should appeal to scientists and statisticians who are relatively new to the field and to others interested in a very readable exploration of the topics covered.*

—*Journal of Computational Intelligence in Finance*

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