The Arnoldfest
Proceedings of a Conference in Honour of V. I. Arnold for his Sixtieth Birthday

Edward Bierstone
Boris Khesin
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Proceedings of a Conference in Honour of V. I. Arnold for his Sixtieth Birthday
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American Mathematical Society
Providence, Rhode Island
The Fields Institute
for Research in Mathematical Sciences

The Fields Institute is named in honour of the Canadian mathematician John Charles Fields (1863–1932). Fields was a visionary who received many honours for his scientific work, including election to the Royal Society of Canada in 1909 and to the Royal Society of London in 1913. Among other accomplishments in the service of the international mathematics community, Fields was responsible for establishing the world's most prestigious prize for mathematics research—the Fields Medal.

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## Contents

Preface ............................................ xi

Program of the Conference .................. xiii

Photographs ..................................... xvi

From Hilbert’s Superposition Problem to Dynamical Systems
V. I. ARNOLD ................................. 1

Recollections
JÜRGEN MOSER ................................. 19

Symplectization, Complexification and Mathematical Trinities
V. I. ARNOLD ................................. 23

Topological Problems in Wave Propagation Theory and Topological
Economy Principle in Algebraic Geometry
V. I. ARNOLD ................................. 39

Geometry and Control of Three-Wave Interactions
MARK S. ALBER, GREGORY G. LUTHER, JERROLD E. MARSDEN
and JONATHAN M. ROBBINS ............... 55

Standard Basis Along a Samuel Stratum, and Implicit Differentiation
EDWARD BIERSTONE and PIERRE D. MILMAN .... 81

A Global Weighted Version of Bezout’s Theorem
JAMES DAMON .................................. 115

Real Enriques Surfaces Without Real Points and
Enriques-Einstein-Hitchin 4-Manifolds
ALEXANDER DEGTYAREV and VIATCHESLAV KHALAMOV .... 131

On the Index of a Vector Field at an Isolated Singularity
W. EBELING and S. M. GUSEIN-ZADE .......... 141

The Exponential Map on $\mathcal{D}^*_\mu$
DAVID G. EBIN and GÉRARD MISIOLEK .... 153

Zeldovich’s Neutron Star and the Prediction of Magnetic Froth
MICHAEL H. FREEDMAN ........................ 165
## Contents

Arnold Conjecture and Gromov-Witten Invariant for General Symplectic Manifolds  
KENJI FUKAYA and KAORU ONO  

Multiplicity of a Zero of an Analytic Function on a Trajectory of a Vector Field  
ANDREI GABRIELOV  

Singularity Theory and Symplectic Topology  
ALEXANDER B. GIVENTAL  

On Enumeration of Meromorphic Functions on the Line  
V. V. GORYUNOV and S. K. LANDO  

Pseudoholomorphic Curves and Dynamics  
H. HOFER and E. ZEHNDER  

Bifurcation of Planar and Spatial Polycycles: Arnold’s Program and Its Development  
YU. S. ILYASHENKO and V. Yu. KALOSHIN  

Singularity Which Has No M-Smoothing  
V. M. KHARLAMOV, S. Yu. OREVKOV and E. I. SHUSTIN  

Symplectic Geometry on Moduli Spaces of Holomorphic Bundles over Complex Surfaces  
BORIS Khesin and ALEXEI ROSLY  

Newton Polyhedra, a New Formula for Mixed Volume, Product of Roots of a System of Equations  
A. KHOVANSKII  

Interactions of Andronov-Hopf and Bogdanov-Takens Bifurcations  
WILLIAM F. LANGFORD and KAIJUN ZHAN  

Solutions of the qKZB Equation in Tensor Products of Finite Dimensional Modules Over the Elliptic Quantum Group $E_r, \eta, sl_2$  
E. MUKHIN and A. VARCHENKO  

Schroedinger Operators on Graphs and Symplectic Geometry  
S. P. NOVIKOV  

On the Dominant Fourier Modes in the Series Associated with Separatrix Splitting for an A-Priori Stable, Three Degree-of-Freedom Hamiltonian System  
MICHAEL RUDNEV and STEPHEN WIGGINS
Homology of $i$-Connected Graphs and Invariants of Knots, Plane Arrangements, etc.  
V. A. VASSILIEV  
451

On Arnold’s Variational Principles in Fluid Mechanics  
V. A. VLADIMOVOV and K. I. ILIN  
471

On Functions and Curves Defined by Ordinary Differential Equations  
SERGEI YAKOVENKO  
497

Global Finiteness Properties of Analytic Families and Algebra of Their Taylor Coefficients  
Y. YOMDIN  
527
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Preface

The Arnoldfest – a conference to celebrate the work of Vladimir Arnold on the occasion of his sixtieth birthday – took place at the Fields Institute in Toronto from June 15 to 21, 1997. The highlight of the meeting was Arnold’s own talks in the Institute’s Distinguished Lecturer program. This volume contains notes of Arnold’s lectures and invited papers by participants in the Arnoldfest.

The Arnoldfest and a Workshop on Symplectic Geometry held the following week were the final major activities during a Program on Singularity Theory and Geometry at the Fields Institute from January to June, 1997. The contributors to this volume include main speakers in the Arnoldfest and the Workshop on Symplectic Geometry, mathematicians and scientists who had been invited to speak but were unable to attend, and participants in the Singularity Theory and Geometry Program.

The very broad spectrum of the articles here reflects the scope of Arnold’s interests. We could not easily classify the articles into groups, so that, apart from the notes of Arnold’s three lectures, the papers are listed in alphabetical order. The one exception is Jurgen Moser’s historical Recollections on Arnold’s first lecture, that we have presented immediately following the notes of the latter. Arnold’s lectures were as provocative as usual, and stimulated some strong discussion! Moser’s Recollections were a welcome contribution.

Arnold was awarded an honorary degree, Doctor of Science, honoris causa by the University of Toronto at a convocation ceremony on June 9, 1997 (three days before his birthday). We have reproduced some photos taken during both the convocation ceremony and the Arnoldfest. We have also included the program of lectures in the Arnoldfest.

A participant in the Arnoldfest could not avoid being struck by the manifestations not only of esteem but also of warm affection for Arnold by his colleagues and former students. These feelings were communicated in part by stories about Arnold told by some of the lecturers. In most cases, the speakers chose not to include these stories in their articles, but we hope that this volume communicates some of the spirit of the Arnoldfest.

Among the features that made the conference a success were its very active poster sessions, the related activities in the Singularity Theory and Geometry Program, and an informal workshop on Applications of Arnold Stability.

The Fields Institute provided excellent facilities and a marvellous ambience for the Arnoldfest. We would like to thank Debbie Iscoe, Karen Walker and Alessia Zucalca for their amiable help in organizing the conference and in preparing this volume. Funding for the Arnoldfest and the Program on Singularity Theory and Geometry was generously provided by the Fields Institute (and, through it,
the Natural Sciences and Engineering Research Council of Canada, and the Ontario Ministry of Education). We are grateful for financial assistance also from the University of Toronto, from the California Institute of Technology for some U.S. participants, and from other institutions that helped support participants.

Edward Bierstone, Boris Khesin, Askold Khovanskii and Jerry Marsden  
June, 1999

Organizing Committees

The Arnoldfest, Conference in Honour of Vladimir Arnold, June 15–21, 1997. Jerrold Marsden, California Institute of Technology (Chair), Edward Bierstone, University of Toronto, John Chadam, University of Pittsburgh, Askold Khovanskii, University of Toronto, Robert Moody, University of Alberta, Tudor Ratiu, University of California at Santa Cruz, William Shadwick, Alexander Varchenko, University of North Carolina at Chapel Hill, Victor Vassiliev, Steklov Mathematical Institute, Moscow.


Informal Workshop on Applications of Arnold Stability, June 13–14, 1997. Ted Shepherd, University of Toronto, Vladimir Vladimirov, Hong Kong University of Science and Technology.
Conference in Honour of Vladimir Arnold
The Fields Institute
June 15–21, 1997

Program

Monday, June 16, 1997

A. Varchenko, University of North Carolina at Chapel Hill
Elliptic Quantum Groups and the Bethe Ansatz

M. E. Kazarian, Steklov Mathematical Institute
Functions on the Circle: Singularities and Invariants

Y. Eliashberg, Stanford University
Invariants of Legendrian Knots via Contact Homology

M. Golubitsky, University of Houston
Animal Gaits and Coupled Cell Systems

J. E. Marsden, California Institute of Technology
The Dynamics and Geometry of Nonholonomic Mechanics

FIELDS INSTITUTE DISTINGUISHED LECTURE SERIES

V. I. Arnold, Steklov Mathematical Institute and Université de Paris-Dauphine
From the Hilbert Superposition Problem to Dynamical Systems

Tuesday, June 17, 1997

R. V. Moody, University of Alberta
Selfsimilarity and Diffraction in Quasicrystals

David Ebin, State University of New York at Stony Brook
Motion of Perfect Fluids with Surface Tension

John Harnad, Concordia University/CRM, Université de Montréal
Loop Groups, R-Matrices and Liouville-Arnold Integrations

P. Holmes, Princeton University
Constrained Euler Buckling

J. W. Milnor, SUNY at Stony Brook
Complex Dynamics
Wednesday, June 18, 1997

E. Bierstone, University of Toronto
Resolution of Singularities

R. de la Llave, University of Texas
Renormalization and Smooth Conjugacy

S. Wiggins, California Institute of Technology
Arnold Diffusion: Issues and Recent Results

V. A. Vassiliev, Steklov Mathematical Institute
Homology of i-Connected Graphs and Invariants of Knots, Plane Arrangements, etc.

V. I. Yudovich, Rostov State University
Dynamical Systems with Cosymmetries and Invariant Pfaffian Forms

FIELDS INSTITUTE DISTINGUISHED LECTURE
V. I. Arnold
Polymathematics: Symplectization, Complexification, and So On

Thursday, June 19, 1997

K. Moffatt, Isaac Newton Institute
Relaxation Under Topographical Constraints

V. Vladimirov, The Hong Kong University of Science and Technology
Hydrodynamic Equilibrium States: Stability

S. Tremaine, University of Toronto
The Stability of the Solar System: An Astronomer’s View

T. Ratiu, University of California at Santa Cruz
Current affiliation: École Polytechnique Fédérale de Lausanne, Switzerland
Reduction by Stages

S. Smale, City University of Hong Kong
Great Problems

Friday, June 20, 1997

P. Slodowy, Universität Hamburg
On Simple Singularities and Lie Groups

D. Fuchs, University of California at Davis
Deformations of Lie Algebras
Program

A. Khovanskii, University of Toronto
Newton Polyhedra, a New Formula for Mixed Volume, Product of Roots of a System of Equations

Y. S. Ilyashenko, Moscow State University
Bifurcation of Planar and Spatial Polycycles: Arnold’s Program and its Development

B. Khesin, University of Toronto
Holomorphic Linking Number and Gauge Groups on Complex Manifolds

FIELDS INSTITUTE DISTINGUISHED LECTURE

V. I. Arnold
Topologically Necessary Wavefront Singularities, the Sturm-Hurwitz Theorem on Fourier Series and Higher Derivative Extensions of Morse (Conley-Zehnder-Chekanov-Chaperon-Floer) Theory

Saturday, June 21, 1997

A. Givental, University of California at Berkeley
Singularity Theory and Symplectic Topology

A. Vershik, Russian Academy of Sciences
Three Dimensional Combinatorics: Statistical Approach

Y. Yomdin, Weizmann Institute of Sciences
Algebraic Differential Equations: Geometry of Solutions and of the First Return Mapping and Algebra of Taylor Coefficients

O. Bogoyavlenskij, Queens' University
The Lie Algebraic Criteria for the Arnold and Komogorov Non-Degeneracy of the Integrable Hamiltonian Systems

A. Neishtadt, Space Research Institute, Russia
On Stability Loss Delay for Dynamical Bifurcations
From left to right: Askold Khovanski; Honourary degree recipients, Sir Martin Rees and Vladimir Arnold; Wendy Cecil-Cockwell, Chairman of the Governing Council

Arnoldfest Speakers

Elya and Vladimir Arnold

Arnoldfest Participants
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The Arnoldfest
Proceedings of a Conference in Honour of V. I. Arnold for his Sixtieth Birthday
Edward Bierstone, Boris Khesin, Askold Khovanskii, and Jerrold E. Marsden, Editors

This volume presents articles originating from invited talks at an exciting international conference held at The Fields Institute in Toronto celebrating the sixtieth birthday of the renowned mathematician, Vladimir Arnold. Experts from the world over—including several from “Arnold’s school”—gave illuminating talks and lively poster sessions. The presentations focussed on Arnold’s main areas of interest: singularity theory, the theory of curves, symmetry groups, dynamical systems, mechanics, and related areas of mathematics.

The book begins with notes of three lectures by V. Arnold given in the framework of the Institute’s Distinguished Lecturer program. The topics of the lectures are:

• From Hilbert’s Superposition Problem to Dynamical Systems
• Symplectization, Complexification, and Mathematical Trinities
• Topological Problems in Wave Propagation Theory and Topological Economy Principle in Algebraic Geometry

Arnold’s three articles include insightful comments on Russian and Western mathematics and science. Complementing the first is Jurgen Moser’s “Recollections”, concerning some of the history of KAM theory.

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