Operator Theory and Its Applications

A. G. Ramm
P. N. Shivakumar
A. V. Strauss
Editors

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Editors
The Fields Institute
for Research in Mathematical Sciences

The Fields Institute is named in honour of the Canadian mathematician John Charles Fields (1863–1932). Fields was a visionary who received many honours for his scientific work, including election to the Royal Society of Canada in 1909 and to the Royal Society of London in 1913. Among other accomplishments in the service of the international mathematics community, Fields was responsible for establishing the world’s most prestigious prize for mathematics research—the Fields Medal.

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Preface

This volume contains selected papers presented at the International Conference on Operator Theory and Its Applications which was held in Winnipeg, in 1998, Oct. 7-11. The Conference was organized by A.G. Ramm, P.N. Shivakumar and A.V. Strauss. Professor A.V. Strauss has died on Oct. 18, 1999. This volume is dedicated to him and a brief note about his life is included.

The Conference was supported by NSF, the Canadian Mathematical Society, the University of Manitoba, the Institute of Industrial Mathematical Sciences, the Fields Institute and other institutions. The organizers and the participants are very thankful to all of the above institutions for their support.

The scientific program of the Conference was rather broad and the applications of the operator theory were strongly emphasized. This is seen in the table of contents of the volume: together with the papers on the abstract operator theory there are many papers on the theory of differential operators, boundary value problems, and on applications to biology, chemistry, wave propagation, inverse problems and many other areas.

The editors believe that applications of operator theory are as important as the theory itself. The operator theory is the language of the modern analysis and its applications and the papers in this volume illustrate this.

A.G. Ramm
P.N. Shivakumar
A.V. Strauss*
Editors

January 2000
Abraham V. Strauss

Professor Abraham Vilgelmovich Strauss was born on June 15, 1920 in Tashkent, the capital of Uzbekistan, a republic of the former USSR. His father, a citizen of Austria, was sent by the Russian government to Tashkent during World War I. After the war, the Strauss family decided to settle down in Peremyshl, a town in Poland where Abraham’s father was born. Abraham grew up in Peremyshl. In 1939, right before the second World War began, Abraham entered Lvov university as an undergraduate student of mathematics. Abraham attended lectures of Banach, Mazur, Schauder, Orlicz, and M. Krein. During the war, from 1941 till 1944, Abraham studied at the Kuibyshev pedagogical institute (Kuibyshev is now called Samara). After graduation, he began working at the Kuibyshev Aviation institute as an instructor in 1944, and was admitted as a Ph.D. student at the Kuibyshev pedagogical institute in 1945 under the supervision of S.P. Pulkin.

Abraham earned both a Ph.D. and a Doctor of Science degree in Mathematics from Moscow State University in 1948 and 1960, respectively. His dissertations have been approved by A. Plessner, M. Naimark, Yu. Berezansky and M. Livsic.

The principal areas of his research were spectral theory of linear operators in Hilbert spaces, extension theory for symmetric linear operators, theory of the characteristic functions and functional models of linear operators, boundary value problems with boundary conditions depending on spectral parameter.

Professor A.V. Strauss solved difficult mathematical problems: he gave a description of all generalized spectral functions and generalized resolvents of symmetric linear operators, not necessarily densely defined, with possibly infinite defect...
indices; he gave a convenient definition of the characteristic operator-valued functions of unbounded non-selfadjoint operators and considered multiplication and factorization theorems for these functions; he established connections between the characteristic functions and generalized resolvents. His definition of the characteristic function allowed him to compute, for the first time, characteristic functions of partial differential operators. This definition contained the definition of the boundary value spaces which are widely used in spectral theory. Professor Strauss published many results in this area. In the theory of extensions of symmetric operators the results of von Neumann, Friedrichs, Krein, Naimark and Strauss are well known.

In 1948, Professor A.V. Strauss began working at the Ul’yanovsk Pedagogical University and from 1954 until his death he worked as a chairman of the Mathematical Analysis Department at this University. More than 35 students received their Ph.D.s in mathematics under his supervision. He was a member of the Presidium of the Scientific and Pedagogical Council of the Ministry of higher education of the USSR, a member of the council on mathematical sciences of the ministry of education of Russian Federation, a member of the Presidium of the regional consortium of the mathematics departments of the pedagogical universities. He was awarded an order of the “Sign of Honour” and some medals. He was a founder and editor-in-chief of the Functional Analysis - Inter-Institutional Collection of Scientific Articles.

This Journal was published by the Ul’yanovsk Pedagogical University and was popular among specialists in operator theory in the former USSR. In the Mathematical Reviews (51#1317) the first issue of this Journal was named “A collection of articles edited by A.V. Strauss”.

Professor A.V. Strauss was an Honoured Scientist of the Russian Federation and a member of the International Academy of Informatization associated with the United Nations.


Professor A.V. Strauss was married to Antonina Yakovlevna Oparina in 1944. She was a school teacher who taught Russian language and literature. She is now retired. Professor Strauss and his wife have two sons, Vladimir and Leonid. Both of them earned a Ph.D in mathematics. Vladimir is a professor of mathematics at the Simon Bolivar University in Caracas. Leonid is Dean of Educational Faculty and an associate professor of mathematics of the Algebra Division of the Faculty of Mechanics and Mathematics of the Ul’yanovsk State University.

Professor Strauss loved music, visual arts and poetry, especially the poetry of A. Pushkin, S. Esenin and A. Mitzkevich. He liked swimming, ping pong, and chess. Professor Strauss was fluent in English, German, French, Polish and knew Latin very well.

Professor A.V. Strauss died on Oct. 18, 1999.

He was an outstanding mathematician and teacher, a very gentle and gracious person. His students and colleagues were very fond of him. Many of his colleagues, as well as people who knew him, will remember him as a wonderful person, an outstanding scientist, and a dedicated teacher.
The editors are very grateful to the wife of A.V. Strauss, his son, Dr V. Strauss, and to Drs. M. Chugunova, A. Vernik and E. Tsekanovsky for the comments and information they provided.

The editors
Publications of Abraham V. Strauss

36. All-union seminar of algebra and number theory tutors from pedagogical institutes. - Matematika v shcole, 1974, N1, pp.94-95 (Russian).
37. From experience of work according to the new program of mathematical analysis course. - Materialy seminara zav. kafedrami matematiki pedinstytutov RSFSR, Tula, 1975, pp.29-33 (Russian).
Publications of Abraham V. Strauss

59. Symmetric relations and module space. - Integral Equations Operator Theory, 1994, 18, 154-165 (Co-authors A. Dijksma, H.S.V. de Snoo)

60. Spectral representations and spectral functions of symmetric operators. - International Conference. Workshop on Applications of Operator Theory. Institute of Industrial Mathematical Sciences, University of Manitoba, 1994, pp.76-77


Operator Theory and Its Applications
A. G. Ramm, P. N. Shivakumar, and A. B. Strauss, Editors

This volume contains a selection of papers presented at an international conference on operator theory and its applications held in Winnipeg. The papers chosen for this volume are intended to illustrate that operator theory is the language of modern analysis and its applications. Together with the papers on the abstract operator theory are many papers on the theory of differential operators, boundary value problems, inverse scattering and other inverse problems, and on applications to biology, chemistry, wave propagation, and many other areas.

The volume is dedicated to the late A. V. Strauss, whose principal areas of research were spectral theory of linear operators in Hilbert spaces, extension theory for symmetric linear operators, theory of the characteristic functions and functional models of linear operators, and boundary value problems with boundary conditions depending on spectral parameter. The bibliography of publications by A. V. Strauss combined with the papers from the conference provide both historical perspective and contemporary research on the field of operator theory and its applications.