



# FIELDS INSTITUTE COMMUNICATIONS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

## Numerical Methods and Stochastics

T. J. Lyons  
T. S. Salisbury  
Editors



**American Mathematical Society**



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Providence, Rhode Island

## The Fields Institute for Research in Mathematical Sciences

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## Preface

This volume contains the proceedings of the Workshop on Numerical Methods and Stochastics, held at the Fields Institute for Research in Mathematical Sciences, April 20-23, 1999. The workshop was part of a thematic program at the Fields Institute, on Probability and its Applications. The workshop featured talks by Dan Crisan, Pierre Del Moral, Jessica Gaines, Alice Guionnet, Terry Lyons, Laurent Miclo, Philip Protter, Frederi Viens, and John Walsh, many of which appear in some form in the present volume. The workshop was preceded by a one-day Symposium on Numerical Stochastics in Finance, on April 19, 1999, which included talks by Phelim Boyle, Pierre L'Ecuyer, Dietmar Leisen, and Philip Protter. The editors would like to thank all the speakers from both the workshop and symposium for a stimulating week of lectures.

The workshop focused on novel approaches to computational problems, based on the latest technology from the theory of probability and stochastic processes. The emphasis was not on a single set of techniques, and the reader will see both particle systems approaches and stochastic analysis represented among the papers. Rather the goal was to identify ideas emerging from probability that should influence future work in both probability and numerical computation.

The editors would like to thank the Fields Institute and its staff for their hospitality during the workshop and symposium. Particular thanks are due to Alesia Zuccala and Debbie Iscoe, who guided and prepared the proceedings manuscript.

Terry Lyons  
Tom Salisbury

July 2002

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This volume represents the proceedings of the Workshop on Numerical Methods and Stochastics held at The Fields Institute in April 1999. The goal of the workshop was to identify emerging ideas in probability theory that influence future work in both probability and numerical computation. The book focuses on new results and gives novel approaches to computational problems based on the latest techniques from the theory of probability and stochastic processes.

Three papers discuss particle system approximations to solutions of the stochastic filtering problem. Two papers treat particle system equations. The paper on “rough paths” describes how to generate good approximations to stochastic integrals. An expository paper discusses a long-standing conjecture: the stochastic fast dynamo effect. A final paper gives an analysis of the error in binomial and trinomial approximations to solutions of the Black-Scholes stochastic differential equations.

The book is intended for graduate students and research mathematicians interested in probability theory.

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