



# FIELDS INSTITUTE COMMUNICATIONS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

## Dynamical Systems and Their Applications in Biology

Shigui Ruan  
Gail S. K. Wolkowicz  
Jianhong Wu  
Editors



**American Mathematical Society**

Dynamical Systems  
and Their Applications  
in Biology

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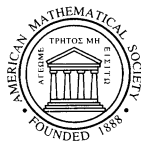
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**American Mathematical Society**  
Providence, Rhode Island

## The Fields Institute for Research in Mathematical Sciences

The Fields Institute is named in honour of the Canadian mathematician John Charles Fields (1863–1932). Fields was a visionary who received many honours for his scientific work, including election to the Royal Society of Canada in 1909 and to the Royal Society of London in 1913. Among other accomplishments in the service of the international mathematics community, Fields was responsible for establishing the world's most prestigious prize for mathematics research—the Fields Medal.

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## Preface

This volume contains the Proceedings of the International Workshop on Dynamical Systems and their Applications in Biology that was held at the Canadian Coast Guard College, August 2–6, 2001, on Cape Breton Island, Nova Scotia.

This workshop was a natural follow up to the International Conference on Differential Equations with Applications to Biology that was held at Dalhousie University July 25–29, 1997. The proceedings of that conference appeared as Volume 21 in this series.

The aim of the workshop was to bring together a very focussed group of international experts to present their latest results in a setting that would also encourage informal discussion of promising research directions in dynamical systems and mathematical biology and promote future research collaborations. As well, we hoped to provide a wonderful opportunity for young Canadian researchers and graduate students to communicate with the leading researchers and to see new and developing areas of research.

Using the renowned meetings at Oberwolfach as a prototype, we chose the Canadian Coast Guard College because of its beautiful setting on Cape Breton Island and because all of the participants could live on the premises and work, eat, and relax together.

There were no parallel sessions and all 40 talks were 30 minutes. Hal Smith of Arizona State University was selected as the AARMS lecturer. He gave a very stimulating talk on competition for two or more essential resources in the chemostat.

There were two very lively panel discussions held in the evenings. One, on promising directions for research in Mathematical Ecology, was lead by Jim Cushing, Herb Freedman, and Hal Smith. The other, on promising directions for research in Mathematical Modeling in Epidemiology, was lead by Fred Brauer and Pauline van den Driessche.

One afternoon was left free from formal talks. Participants were encouraged to explore Cape Breton Island in small groups. This provided an excellent opportunity for the more junior researchers and graduate students to get to know the more senior ones in very pleasant and informal surroundings.

The participants were truly an international group of experts. Besides participants from all parts of Canada, there were participants from Austria, China, Japan, Spain, Taiwan, the United Kingdom, and the United States.

This proceedings contains 19 papers by 35 authors and covers articles involving the qualitative and/or numerical analysis of models involving ordinary, partial, functional and stochastic differential equations as well as difference equations. Applications include epidemiology, populations dynamics, and physiology.



The workshop was generously supported by the National Program Committee of the Institutes (The Fields Institute for Research in Mathematical Sciences in Toronto, Le centre de recherches mathématiques (CRM) in Montréal, and the Pacific Institute for Mathematical Sciences (PIMS)), by Dalhousie University (Vice-President of Academic and Research, Research Services and Graduate Studies) and by the Atlantic Association for Research in Mathematical Sciences (AARMS). The Canadian Coast Guard College provided an ideal setting and in the typical maritime style, the staff went out of their way to make everyone's stay as comfortable and enjoyable as possible. We also wish to thank Debbie Iscoe, the Publications Manager of The Fields Institute, for her professional assistance.

Shigui Ruan  
Gail Wolkowicz  
Jianhong Wu  
September 2002

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This volume is based on the proceedings of the International Workshop on Dynamical Systems and their Applications in Biology held at the Canadian Coast Guard College on Cape Breton Island (Nova Scotia, Canada). It presents a broad picture of the current research surrounding applications of dynamical systems in biology, particularly in population biology.

The book contains 19 papers and includes articles on the qualitative and/or numerical analysis of models involving ordinary, partial, functional, and stochastic differential equations. Applications include epidemiology, population dynamics, and physiology.

The material is suitable for graduate students and research mathematicians interested in ordinary differential equations and their applications in biology.

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