Galois Theory, Hopf Algebras, and Semiabelian Categories

George Janelidze
Bodo Pareigis
Walter Tholen
Editors
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Editors
The Fields Institute
for Research in Mathematical Sciences

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Preface

During the week of September 23–28, 2002, the editors of this volume organi-
zied a Workshop on Categorical Structures for Descent and Galois Theory, Hopf
Algebras, and Semiabelian Categories at the Fields Institute for Research in Math-
ematical Sciences in Toronto. The goal of the Workshop was to bring together
researchers working in the quite distinct but nevertheless interrelated and partly
overlapping areas mentioned in its title. The meeting was attended by almost
eighty mathematicians from various research communities and boasted twenty in-
vited lectures and numerous contributed talks that led to an inspiring atmosphere
of learning and scientific exchange.

This volume can only partially reflect the Workshop’s themes but should nev-
evertheless give the reader a good idea about the current connections among abstract
Galois theories, Hopf algebras, and semiabelian categories. Here is a very brief
indication of the origins of those connections. Hopf algebras arrived to the Galois
theory of rings as early as the 1960s — independently of, but in fact similarly to,
the way in which algebraic group schemes were introduced to the theory of étale
coverings in algebraic geometry. Galois theory, in turn, was extended to elemen-
tary toposes and was then formulated in purely categorical contexts. Eventually it
became general enough to even include abstractions of the theory of central exten-
sions, to mention only one of various fairly recent developments. In fact, classically,
central extensions of groups together with the homology functors $H_1(-, \mathbb{Z})$ and
$H_2(-, \mathbb{Z})$ can be used to begin homological algebra, just like covering spaces to-
gether with the homotopy functors $\pi_0$ and $\pi_1$ are the starting gadgets of homotopy
theory. Finally, during the past four years semiabelian categories have emerged as
a very good environment in which to pursue not just basic modern algebra but in
fact homological algebra of groups and other non-abelian structures categorically.

Given the diversity of the backgrounds of the presenters at the Workshop, this
volume cannot be expected to contain a homogeneous sequence of chapters on its
themes. Rather, the reader will find a collection of beautiful but fairly independent
articles on selected topics in algebra, topology, and pure category theory that should
seriously contribute to the categorical unification of the subjects in question. The
survey articles contained in this volume should be particularly helpful in this regard.

A rough general “map” of the topics/articles presented in this volume may be
displayed as follows, with the numbers referring to the (alphabetical) list of
contributions contained in the volume. Most of the papers are mentioned more
than once. Solid lines represent links explicitly discussed in this volume, while
dotted lines indicate other known links.
1. M. Barr, *Algebraic cohomology: the early days*
2. F. Borceux, *A survey of semi-abelian categories*
3. D. Bourn, *Commutator theory in regular Mal'tsev categories*
4. D. Bourn and M. Gran, *Categorical aspects of modularity*
5. R. Brown, *Crossed complexes and homotopy groupoids as non commutative tools for higher dimensional local-to-global problems*
6. M. Bunge, *Galois groupoids and covering morphisms in topos theory*
7. S. Caenepeel, *Galois corings from the descent theory point of view*
8. B. Day and R. H. Street, *Quantum categories, star autonomy, and quantum groupoids*
10. M. Gran, *Applications of categorical Galois theory in universal algebra*
11. C. Hermida, *Fibrations for abstract multicategories*
12. J. Huebschmann, *Lie-Rinehart algebras, descent, and quantization*
13. P. T. Johnstone, *A note on the semiabelian variety of Heyting semilattices*
14. G. M. Kelly and S. Lack, *Monoidal functors generated by adjunctions, with applications to transport of structure*
15. M. Khalkhali and B. Rangipour, *On the cyclic homology of Hopf crossed products*
16. G. Lukács, *On sequentially h-complete groups*
17. J. L. MacDonald, *Embeddings of algebras*
18. A. R. Magid, *Universal covers and category theory in polynomial and differential Galois theory*
19. N. Martins-Ferreira, *Weak categories in additive 2-categories with kernels*
20. T. Palm, *Dendrotopic sets*
21. A. H. Roque, *On factorization systems and admissible Galois structures*
22. P. Schauenburg, *Hopf-Galois and bi-Galois extensions*
23. J. D. H. Smith, *Extension theory in Mal'tsev varieties*
24. L. Sousa, *On projective generators relative to coreflective classes*
25. J. J. Xarez, *The monotone-light factorization for categories via preorders*
26. J. J. Xarez, *Separable morphisms of categories via preordered sets*
27. S. Yamagami, *Frobenius algebras in tensor categories and bimodule extensions*
We express our sincere thanks to the Fields Institute for hosting and supporting the Workshop and publishing this volume. We are particularly grateful to Ms. Debbie Iscoe for her help in preparing the files. We also thank the Faculty of Arts of York University for additional financial assistance.

George Janelidze
Bodo Pareigis
Walter Tholen
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This volume is based on talks given at the Workshop on Categorical Structures for Descent and Galois Theory, Hopf Algebras, and Semiabelian Categories held at The Fields Institute for Research in Mathematical Sciences (Toronto, ON, Canada). The meeting brought together researchers working in these interrelated areas.

This collection of survey and research papers gives an up-to-date account of the many current connections among Galois theories, Hopf algebras, and semiabelian categories. The book features articles by leading researchers on a wide range of themes, specifically, abstract Galois theory, Hopf algebras, and categorical structures, in particular quantum categories and higher-dimensional structures.

Articles are suitable for graduate students and researchers, specifically those interested in Galois theory and Hopf algebras and their categorical unification.