Asymptotic Methods in Stochastics

Festschrift for Miklós Csörgő

Lajos Horváth
Barbara Szyszkowicz
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Editors
The Fields Institute
for Research in Mathematical Sciences

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Preface

ICAMS’02, an International Conference on Asymptotic Methods in Stochastics was organized and held in honour of the work of Miklós Csörgő on the occasion of his 70th birthday at Carleton University, Ottawa, Canada, 23–25 May 2002. The conference was hosted and sponsored by the Laboratory for Research in Statistics and Probability (LRSP), Carleton University–University of Ottawa, the School of Mathematics and Statistics, Carleton University, and co-sponsored by The Fields Institute for Research in Mathematical Sciences. This international meeting was a smaller version of an earlier conference, ICAMPS’97 (International Conference on Asymptotic Methods in Probability and Statistics), that was held at Carleton University in July 1997. For the proceedings volume of the latter conference we refer to [1].

We are pleased to publish the proceedings of ICAMS’02 in FIELDS INSTITUTE COMMUNICATIONS by AMS, and it is our pleasure to dedicate this collection of research papers to Miklós Csörgő as a token of respect and appreciation of his work in Probability and Statistics by all the contributors to this volume, and all the participants of ICAMS’02. We are grateful to the contributors for submitting their papers for publication in this volume, as well as to the referees for their valuable time and enhancing work on it. All papers have been refereed, and accordingly revised if so requested by the editors. We wish to record here our sincere thanks to everyone for their further time, care and collaboration throughout this elaborate process.

The papers in this volume reflect the wide ranging interest of Miklós in Probability and Statistics, and nearly all of them are connected to his research. The editors also have a 69 page résumé of his work over the past forty or so years, titled PATH PROPERTIES OF FORTY YEARS OF RESEARCH IN PROBABILITY AND STATISTICS: IN CONVERSATION WITH MIKLÓS CSÖRGŐ. This article, together with Miklós’s list of publications is available as No. 400–2004 of the Technical Report Series of LRSP. It can also be accessed on the LRSP website: www.lrsp.carleton.ca, as well as on the Fields Institute website: www.fields.utoronto.ca/publications/supplements/.

Unfortunately, due to space limitations, we could not include this résumé with its 311 references and Miklós’s list of publications in this collection.

More than half of the 28 papers in this volume are up-to-date surveys on various active research areas in Probability and Statistics. All the sections except for Part 2 are headed by survey papers that are also indicative of the main themes of these sections. All other papers, including Part 2 in which there are three survey papers on different themes, are alphabetically ordered.

In Part 1 Csáki, Földes and Shi provide a survey of their joint work with Miklós on path properties of stochastic processes, a most insightful review of their collaboration on strong approximations of local time and additive functionals, path
properties of Cauchy principal values of Brownian local time, iterated processes, level crossings of the empirical process, Vervaat and Vervaat-error processes and Banach space valued stochastic processes. Khoshnevisan presents a self-contained theory of quasi-sure results via Brownian sheet connections. In the first of their two papers in this volume Peccati and Yor provide a unified framework by means of Hardy’s inequality in $L^2[0, 1]$ for two results concerning the existence of certain integrals associated with a one dimensional Brownian motion starting from zero and the principal values of Brownian local times. In the second of their two papers Peccati and Yor generalize and give new proofs of four limit theorems on quadratic functionals of Brownian motion and Brownian bridge that were recently obtained by Deheuvels and Martynov, and establish explicit connections with occupation times of Bessel processes, Poincaré’s Lemma and the class of quadratic functionals of Brownian local times studied in their preceding paper. Estimating the local time of a Wiener process from its values at integers, Révész provides a new look at one of his results of twenty years ago with Miklós.

The papers in Part 2 survey several new directions in probability theory and its applications. Bhansali, Holland and Kokoszka study properties of chaotic maps that provide non-linear, non-Gaussian models as alternatives to earlier established linear and Gaussian stochastic models for the class of discrete-time long-memory stationary processes. Davydov and Paulauskas survey recent results on, and give a short introduction to, $p$-stable convex compact sets in Banach spaces, with special attention to stable random zonotopes. Davydov and Zitikis survey results on convex rearrangements, called by them convexifications, of stochastic processes. They also provide a view of relationships of convexifications with the operators of monotone and convex rearrangements in functional analysis and with the generalized Lorenz curves of econometrics. Dawson, Gorostiza and Wakolbinger survey recent work on hierarchical random walks with emphasis on transience-recurrence phenomena and in particular on the notion of degree of transience. They also describe a family of hierarchical random fields. Applications of hierarchical random walks and fields in statistical physics and branching processes are also discussed. Studying the expected distance from the origin after $n$ steps of the so-called isotropic Pearson random walk in the plane, Ross and Shao improve the upper bound on Helgason’s number.

The classical Erdős–Hsu–Robbins notion of complete convergence has led to various extensions of this original idea. One of these extensions is Heyde’s notion of what is now called precise asymptotics. Following their extensive survey of precise asymptotics for sums, in Part 3 Gut and Steinebach extend these results to renewal counting processes and first passage time processes of random walks. As an analog of Heyde’s theorem for ordinary means, Sándor Csörgő obtains the precise asymptotic behaviour of bootstrap means.

In Part 4, Ćwiklińska and Rychlik present necessary and sufficient conditions for the weak convergence of random sums and maximum random sums of independent random variables. Tomkins concludes necessary and sufficient conditions for the almost sure and complete stability of weighted maxima of bounded i.i.d. random variables.

Part 5 is devoted to change-point analysis. Hušková contributes a survey of procedures based on permutation tests or resampling methods for obtaining approximations to the critical values of various test procedures for detecting changes in statistical models. Aly proposes and studies $L$–statistics based test procedures for
detecting a change in the distribution of a random sample. Atanafu and Gombay define truncated sequential tests via the generalized likelihood ratio to detect change in observations described by the nested random effects model. Orasch studies the asymptotic behaviour of U-statistics based processes whose appropriate functionals can be used to detect multiple changes in the distribution of a sample of possibly vector valued observations.

In Part 6, based on quantiles, comparison distributions, and conditional quantiles, Parzen develops a unified non-parametric framework which he calls “Statistical methods learning: for understanding and applying statistical methods.” Using a strong martingale approach to weak convergence, Burke considers cumulative sum processes that can be used to test the fit of models in multivariate regression and the proportional hazards model of survival analysis. Given certain marginals, Dabrowski and Dehling study the conditional distribution of a multinomial sample and obtain a local multivariate normal limit theorem. As a consequence they prove asymptotic normality of the so-called H-coefficient in certain nonparametric unfolding models with dichotomous data. Ghoudi and Rémillard, continuing their work that was published in the above mentioned volume [1], provide a unified treatment of inference procedures that are based on pseudo-observations in the multivariate setting, and give several examples of applications as well.

Part 7, devoted to applications to economics, opens with a review of recent advances in the probabilistic and statistical theory of GARCH and related processes by Berkes, Horváth and Kokoszka. GARCH type models are extensively used in modeling returns on speculative assets. Kulperger’s paper deals with aspects of contingent claim pricing in the incomplete discrete time model for returns via seeking a method to choose amongst members of the family of risk neutral measures that is close in some sense to the historical model measure. McLeish builds on using high and low price records of financial time series for estimating volatility parameters and correlation, and finds a multivariate normal approximation to the joint distributions of high, low and close price records to be a useful tool for pricing certain path-dependent options. Yu in his paper surveys recent developments on asymptotic results for residual processes of (G)ARCH time series models, and shows that, though most common processes such as partial sums and empirical processes have Gaussian limits that depend on the unknown parameters of these models, some of these processes when properly normalized will have a Gaussian limit that is free of model parameters. Hence one can, for example, test for model fitness or model misspecification in such situations.

In Part 8, Csörgö, Szyszkowicz and Wang survey weighted approximations in probability and strong limit theorems for self-normalized partial sums processes. In the last paragraph of our above mentioned résumé in conversation with Miklós Csörgö (cf. [2]), we mention a number of further important references on invariance principles in this regard. In the second paper of Part 8, Wang sharpens an earlier result on a Darling-Erdős type theorem for self-normalized sums.

We now wish to take this opportunity to sincerely thank the Natural Sciences and Engineering Research Council (NSERC) of Canada for their financial support of our LRSP by their Major Facility Access (MFA) awards in these years. Without these MFA awards, it would have been impossible for our LRSP to exist, and for us to even think about organizing our ICAMS’02. We are also grateful to the Fields Institute for Research in Mathematical Sciences for their financial support of our
conference. We hope very much that this volume and the international success of ICAMS’02 will have contributed to the justification of their trust in us.

Last, but not least, we most sincerely wish to thank Gillian Murray, the Coordinator of our manifold LRSP activities, for her help throughout, for taking care of all the logistics of ICAMS’02, and for her invaluable technical skill and role in preparing this volume for publication.

In conclusion, we also want to express our appreciation to the Editorial Board of the Fields Institute for their approval of the publication of these proceedings in their Communications Series, to Carl R. Riehm, the Managing Editor of Publications, and Tom Salisbury, the Deputy Director, for their kind attention to, and sincere interest in, the publication of this volume, and to their Publications Manager, Debbie Iscoe, for her cooperation and expert help in its preparation for the AMS publishers. We hope very much that the readers will find this collection of papers informative and also helpful in their studies and work.

LAJOS HORVÁTH
BARBARA SZYSZKOWICZ

References


This volume, honoring over forty years of Miklós Csörgő's work in probability and statistics, reflects the state of current research. It offers a comprehensive collection of surveys introducing new results with complete proofs and expository papers giving an historic overview.

Contributions were made by an international cast of experts. The book covers the following topics: path properties of stochastic processes, probability theory with applications, complete convergence of renewal counting processes and bootstrap means, weak convergence of random size sums, almost sure stability of weighted maxima, procedures for detecting changes in statistical models, statistical inference via conditional quantiles, cumulative sums, multinomial samples, empirical processes, applications to economics, and self-normalized partial sums processes. The section, “Applications to Economics”, deals primarily with applications of stochastics to financial time series models.

The book is suitable for graduate students and researchers interested in probability theory, stochastic processes, mathematical statistics, and applications of these mathematical/statistical sciences.