
Luigi Rodino
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Editors
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The Fields Institute for Research in Mathematical Sciences

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Preface

Supported financially by the International Society for Analysis, its Applications and Computation (ISAAC), the Academic Initiative Fund of the Department of Mathematics and Statistics at York University, and a subsidy from the Office of the Vice-President Academic of York University, the ISAAC Workshop on Pseudo-Differential Operators: Partial Differential Equations and Time-Frequency Analysis was held at the Fields Institute from December 11, 2006 to December 15, 2006.

Featured at the workshop were five 2.5-hour mini-courses and twenty-two 45-minute lectures. The mini-courses were designed primarily for graduate students and post-docs. The 45-minute lectures were given by a broad spectrum of speakers from Ph.D. students and post-docs to young professors to experts in a congenial and supportive environment offered by the Fields Institute.

Applications to partial differential equations and geometric analysis have always been at the forefront of mainstream analysis embraced by ideas and techniques in pseudo-differential operators. A very new development in pseudo-differential operators in the last decade has been seen in the interconnections with engineering, geology, atmospherical physics and medical imaging, which can be collectively classified as time-frequency analysis.

This volume contains the five papers based on the mini-courses given by Professors Charles L. Epstein, Peter Greiner, Karlheinz Gröchenig, Luigi Rodino and Bert-Wolfgang Schulze. Together with the fifteen papers on related topics, this volume provides a panorama of great interest in pseudo-differential operators. All papers have been peer-reviewed and topics include partial differential equations, geometric analysis, Fourier integral operators, localization operators, Gabor transforms, wavelet transforms, Rihaczek transforms and time-frequency analysis. A focal point is the relatively new transform known as the Stockwell transform, which is a hybrid of the Gabor transform and the wavelet transform. The mathematical underpinnings, applications and ramifications of this interesting transform are given.

From this permanent record of the ISAAC workshop, it can be envisaged that the future of the subject on pseudo-differential operators will be in mainstream analysis going forward hand in hand with applications and computation in many sectors of modern science and technology.

August 2007
Luigi Rodino, Bert-Wolfgang Schulze and M. W. Wong
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This volume is based on lectures given at the workshop on pseudo-differential operators held at the Fields Institute from December 11, 2006 to December 15, 2006. The two main themes of the workshop and hence this volume are partial differential equations and time-frequency analysis. The contents of this volume consist of five mini-courses for graduate students and post-docs, and fifteen papers on related topics. Of particular interest in this volume are the mathematical underpinnings, applications and ramifications of the relatively new Stockwell transform, which is a hybrid of the Gabor transform and the wavelet transform. The twenty papers in this volume reflect modern trends in the development of pseudo-differential operators.