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Preface

Lie algebras and their representations form an extremely rich and important field of mathematics. This area can sometimes be technically abstract, but is also well known for its connections and applications to many other disciplines such as geometry, topology, mathematical physics, and algebraic combinatorics.

The interplay between algebra and geometry goes both ways. One uses geometric methods to construct or gain insight on representations of Lie groups and Lie algebras. The best known examples in this direction include the Borel-Weil-Bott construction, Springer theory of Weyl group representations, Kazhdan-Lusztig theory for category $\mathcal{O}$, and Nakajima and Lusztig quiver variety realizations of Kac-Moody algebras and their representations. On the other hand, one is often able to use the representation theory of semisimple or affine Lie algebras to organize and better understand the geometric invariants (such as homology) of various interesting varieties, such as Hilbert schemes, flag varieties, and affine Grassmannians. In addition, geometric representation theory has been found to be intimately related to the combinatorics of crystals and quivers.

Within the theory of Kac-Moody algebras, it is the finite-dimensional and affine Lie algebras that stand out, not only for their well-developed representation theory, but also for applications to number theory, combinatorics, and mathematical physics. Extended affine Lie algebras are a class of Lie algebras that encompasses these two important types of Lie algebras, as well as toroidal Lie algebras. The structure theory of extended affine Lie algebras (sometimes jokingly called Canadian algebras by friends) has just recently been completed, as the culmination of the last decade of work of a number of researchers, many of them Canadian.

It was before such a background that the University of Ottawa hosted a summer school in "Geometric Representation Theory and Extended Affine Lie Algebras", on June 15–27, 2009. The summer school was followed by a week-long conference on the same general area of representation theory. There were 3 lecture series in each of the two weeks of summer school, with one and half an hour lecture per weekday. These lecture series were as follows:

(i) *Introduction to geometric representation theory*, by Joel Kamnitzer.
(ii) *Introduction to quantum groups and crystals*, by Seok-Jin Kang.
(iii) *Geometric realizations of crystals*, by Alistair Savage.
(iv) *Nilpotent orbits and finite W-algebras*, by Weiqiang Wang.
(v) *Affine, toroidal and extended affine Lie algebras*, by Erhard Neher.
(vi) *Representation theory of affine and toroidal Lie algebras*, by Vyjayanthi Chari.

In his lectures, Kamnitzer explains three distinct geometric approaches of constructing irreducible rational representations of the Lie group GL_n, namely, the Borel-Weil construction, the Ginzburg construction, and the geometric Satake correspondence. While each of the approaches generalizes to more general semisimple or reductive Lie groups, the focus on type A helps to make things explicit and avoid introducing Langlands dual groups and further deeper geometric objects such as Nakajima quiver varieties.

Kang starts his lectures by reviewing the basics of crystal basis theory. He then continues to describe the theory of perfect crystals which has played an important role in providing a realization of crystals for irreducible integrable modules of quantum affine algebras. In the case of level one integrable modules, a combinatorial approach to crystals using Young walls is explained in the end.

Like the semisimple Lie algebras, extended affine Lie algebras admit root space decompositions. The associated root systems can be understood and studied in the framework of affine reflection systems, which, roughly speaking, consist of finite root systems together with additional extension data. The structure theory of extended affine Lie algebras serves as a good example of the unity of mathematics: One needs all the important classes of non-associative algebras to describe their structure (alternative algebras, Jordan algebras and structurable algebras). Neher's lectures provide a detailed exposition of the structure theory, and they complement Chari's lectures on representations.

Savage's lectures on realizations of crystals are geometric in nature, in contrast to the algebraic approach taken by Kang. The main geometric objects involved are partial flag manifolds, and Lusztig and Nakajima quiver varieties. The crystal graph corresponds to the set of irreducible components of these (or closely related) varieties. Along the way, the basics of quiver representations are also introduced. In the end, Savage describes the relationship between the geometric realization and combinatorial realization via tableaux.

The category of positive level integrable modules of affine Lie algebras with finite-dimensional weight spaces is semisimple and the irreducibles satisfy the Weyl-Kac character formula. Chari's lectures focus on the finite-dimensional level zero modules of the loop algebras. The category of such modules is not semisimple, and homological algebra and quivers have come to play an increasingly important role recently. Some important constructions such as Weyl modules are motivated through their connections to quantum affine algebras.

Finite W-algebras, which appeared as a counterpart to the affine W-algebras in the 1990's, are certain highly nonlinear algebras resulting from a (quantum) hamiltonian reduction. Alternatively, finite W-algebras over the complex numbers can be regarded as a quantization of Slodowy slices, i.e., transversal slices to nilpotent orbits for semisimiple Lie algebras. Finite W-algebras in prime characteristic also arise naturally in Premet's solution to the Kac-Weisfeiler conjecture for modular representations of Lie algebras. Wang's lectures provide an overview of some basic aspects of the structure and representation theory of finite W-algebras and their super generalizations.

More than 130 participants from 14 different countries took part in the summer school. In addition to many Canadian students, a particularly strong contingent of students came from Korea and, of course, the United States. We gratefully acknowledge the generous financial support of The Fields Institute for Research in the Mathematical Sciences (Canada) and the National Science Foundation (USA). We thank the University of Ottawa for providing excellent facilities for the summer school, and all the lecture series speakers for their fine contributions. We also thank all the participants for their active participation which made the summer school (and the workshop) such a success. Finally, we would like to thank the staff of the Fields Institute, particularly Debbie Iscoe, for their help in the preparation of this book.

Erhard Neher
Alistair Savage
Weiqiang Wang

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