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The Fields Institute for Research in Mathematical Sciences

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Research Directions in Number Theory

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The Fields Institute for Research in Mathematical Sciences

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Preface

This volume is a collection of papers on number theory which evolved out of the workshop *WIN—Women In Numbers*, held November 2–7, 2008, at the Banff International Research Station (BIRS) in Banff, Alberta, Canada. It includes articles showcasing outcomes from collaborative research initiated during the workshop as well as survey papers aimed at introducing graduate students and recent PhDs to important research topics in number theory.

The workshop and this volume are part of a broader WIN initiative, whose goals are to raise the profile of active female researchers in number theory and increase their participation in research activities in the field. Although the number of female number theorists has grown over the past 20 years, this group remains underrepresented at high profile conferences and among the tenured faculty at top research universities. This underrepresentation has profound negative consequences on the recruitment and training of future female mathematicians. Part of the purpose of the BIRS workshop and this volume was to address this issue.

The WIN workshop represented a unique effort to combine a top level technical research program with strong broad impact. The workshop brought together senior and junior female researchers in the field of number theory. The main goals were to highlight and increase the research activities of women in number theory, and to train female graduate students in number theory and related fields. Emphasis was placed on on-site collaboration on open research problems as well as student training. Students were introduced to areas of active research in collaborative group projects which connected them with senior female faculty and with a network of future potential collaborators.

The workshop included 41 female number theorists and was organized by the last three editors of this volume. The attendees were spread across all levels of seniority, ranging from graduate students to postdocs and senior researchers. Based on the participants' research interests and expertise, the workshop organizers formed eight research groups each consisting of 4-6 members; generally, this included 2 group leaders who planned the specific topics and 2-4 other members. The content spanned a wide range of topics in arithmetic geometry and algebraic, algorithmic, and analytic number theory. Many of the group projects initiated at the workshop led to new research results that are published in this volume and elsewhere. A follow-up BIRS workshop entitled *WIN-2—Women In Numbers 2* will take place November 6–11, 2011, at BIRS. Other future WIN events are in the planning stage at the time of this volume's publication.

Workshop project titles and lectures

The eight project groups each had one or two group leaders who developed the scientific content and gave introductory lectures for the projects. On the last two days of the conference, the other participants of each group gave presentations on the group's progress reports.

- *Computations on Hilbert modular surfaces*
  - Lecture 1: Helen Grundman

  - Lecture 2: Kristin Lauter
  - Project report: Jennifer Johnson-Leung, Adriana Salerno, Bianca Viray, Erica Wittenborn
- *L-functions and Frobenius distributions*
  - Lecture 1: Chantal David
  - Lecture 2: Alina-Carmen Cojocaru
  - Project report: Alina Bucur, Brooke Feigon, Matilde Lalín
- *Class groups of function fields*
  - Lecture 1: Renate Scheidler
  - Lecture 2: Yoonjin Lee
  - Project report: Lisa Berger, Jing Long Hoelscher, Jennifer Paulhus
- *Computation of pairings on hyperelliptic curves*
  - Lecture 1: Kirsten Eisentraeger
  - Lecture 2: Edlyn Teske
  - Project report: Jennifer Balakrishnan, Juliana Belding, Sarah Chisholm, Katherine Stange
- *Galois covers of curves in characteristic p*
  - Lecture 1: Katherine Stevenson
  - Lecture 2: Rachel Pries
  - Project report: Linda Gruendken, Laura Hall-Seelig, Bo-Hae Im, Ekin Ozman
- *Zeta functions of graphs*
  - Lecture 1: Audrey Terras
  - Lecture 2: Winnie Li
  - Project report: Shabnam Akhtari, Habiba Kadiri, Beth Malmskog, Michelle Manes
- *Modular forms*
  - Lecture 1: Stephanie Treneer
  - Lecture 2: Ling Long
  - Project report: Sharon Garthwaite, Holly Swisher
- *Galois representations*
  - Lecture: Mirela Ciperiani
  - Project report: Margaret Upton, Núria Vila

Contributions in this volume

In 2009, the editors invited WIN workshop participants to submit articles to this proceedings volume. Papers were also solicited from some female number theorists who had been unable to attend. The aim was to document new research which emerged from the BIRS workshop and to encourage the publication of survey papers that showcase active areas of research in number theory. Following a careful and thorough refereeing process by external experts, 16 submissions were accepted, largely in the areas of arithmetic geometry and algebraic number theory.

We grouped the contributions in this volume into five areas. Clusters of papers center around the four topics of moduli spaces and Shimura curves, curves and Jacobians over finite fields, Galois covers of function fields in positive characteristic, and zeta functions of graphs, with a fifth group of three individual articles on modular forms, Iwasawa theory, and Galois representations, respectively.

Moduli spaces and Shimura curves. Arithmetic intersection theory on moduli spaces and Shimura curves is a very active area of recent research in number theory. This volume includes four contributions to this field that focus predominantly on computational aspects of Shimura curves and modular curves and varieties. Grundman's paper *Hilbert modular variety computations* provides a survey of computational methods used in studying Hilbert modular varieties, with particular emphasis on determining numerical invariants used in their classification. Bayer's *Contributions to Shimura curves* presents new results on the computation of the automorphic functions of canonical models for Shimura curves as well as equations defining CM points on such curves. The six-author article entitled *Igusa class polynomials, embeddings of quartic CM fields, and arithmetic intersection theory* by Grundman et al. evolved from research conducted at the WIN workshop; it investigates the arithmetic intersection numbers conjectured by Bruinier and Yang and finds both numerical support for the conjecture and some anomalies in the general case. Finally, Mantovan's contribution *Cohomology of PEL-type Shimura varieties with non-trivial ℓ-adic coefficients* extends previous results on computing ℓ-adic cohomology of Shimura varieties from the case of constant ℓ-adic coefficients to a more general scenario.

Curves and Jacobians over finite fields. Curves over finite fields and their Jacobians have long been objects of intense number theoretic study. Much information can be gleaned from the zeta functions of these objects, and recent decades have seen the emergence of exciting cryptographic applications. All three articles in this area originate from WIN workshop group projects. Pairings on elliptic and hyperelliptic curves have seen a recent surge of research activity. The six-author paper *Pairings on hyperelliptic curves* by Balakrishnan et al. surveys constructions of pairing-friendly hyperelliptic curves and develops a unified framework for all hyperelliptic pairings proposed to date. The four-author contribution *Biased statistics for traces of cyclic p-fold covers over finite fields* by Bucur et al. provides statistics of the trace of Frobenius for a cyclic p-fold cover of the projective line, generalizing previous work by the authors that focused mainly on the case $p = 3$. The paper *The ℓ-rank structure of a global function field* by Berger and four co-authors analyzes in detail the behaviour of the ℓ-rank of the Jacobian associated to any curve over a finite field when the base field is gradually enlarged.

Galois covers of function fields in positive characteristic. There are many open problems related to Galois covers and fundamental groups of curves in positive characteristic. These arise, in large part, because of the phenomena of wild ramification. This area has close connections with the previous topic of curves and Jacobians. The volume contains three papers on this topic. Pries & Stevenson's *A survey of Galois theory of curves in characteristic p* provides a survey on Galois covers and fundamental groups of curves in positive characteristic. An open problem is to determine for a non-abelian quasi-p group G the smallest genus that can occur for a G-Galois cover of the affine line. Bouw's contribution *Covers of the affine line in positive characteristic with prescribed ramification* answers this problem for the case that $G = A_d$ is the alternating group on d letters, with $p + 2 \leq d \leq 2p - 1$, and the six authors of *Semi-direct Galois covers of the affine line* by Gruendken et al. provide the answer for the case $G = (\mathbb{Z}/\ell)^b \rtimes \mathbb{Z}/p$, with $b \in \mathbb{N}$ and ℓ a prime distinct from p. This last paper grew out of research conducted at the WIN workshop.

Zeta functions of graphs. Zeta functions of graphs represent a new area of number theory, with connections to combinatorial and analytic number theory. Three papers on this topic appear here. Terras' *Looking into a graph theory mirror of number theoretic zetas* surveys the Ihara zeta function for irregular graphs and its connections with number theoretic zeta functions. A second survey article by Li entitled *Zeta functions of group based graphs and complexes* focuses on graphs and complexes arising from quotients of the Bruhat-Tits buildings associated to $PGL_2(F)$ and $PGL_3(F)$ for a non-archimedean local field F with finite residue field. The contribution *Ramified covers of graphs and the Ihara zeta functions of certain ramified covers* by Malmkog and Manes has its basis in research conducted at the WIN conference. It revisits the zeta functions discussed by Terras, investigating the Ihara zeta functions of regular graphs as well as certain coverings of these graphs.

Other topics. Three other important areas of number theory are represented by contributions in this volume. Research on modular forms and Eisenstein series conducted at the WIN workshop spawned the four-author paper *Zeros of classical Eisenstein series and recent developments* by Garthwaite et al. In her expository article *On the μ-invariant in Iwasawa theory*, Sujatha reviews this important classical invariant attached to Iwasawa modules. Finally, the contribution entitled *Galois representations and the tame inverse Galois problem* by Arias-de-Reyna and Vila showcases progress made on a variant of the inverse Galois problem over the rational numbers through the analysis of the Galois representations arising from arithmetic-geometric objects.

Workshop website

`http://www.birs.ca/events/2008/5-day-workshops/08w5112`

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- Banff International Research Station
- Fields Institute for Research in Mathematical Sciences
- Microsoft Research
- Pacific Institute for the Mathematical Sciences
- United States National Security Agency
- University of Calgary

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WIN Editorial Committee November 2010

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Kristin Lauter, Microsoft Research, USA
Rachel Pries, Colorado State University, USA
Renate Scheidler, University of Calgary, Canada

WIN—Women In Numbers, November 2008

Workshop Participants and Affiliations at the Time of the Workshop

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Jennifer Balakrishnan, Massachusetts Institute of Technology, USA
Juliana Belding, University of Maryland, USA
Lisa Berger, Stony Brook University, USA
Alina Bucur, Massachusetts Institute of Technology, USA
Sarah Chisholm, University of Calgary, Canada
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