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Modular Calabi-Yau Threefolds

Christian Meyer

American Mathematical Society

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The Fields Institute for Research in Mathematical Sciences

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Preface

The proof of the Taniyama-Shimura Conjecture by A. Wiles et al. in the 1990s (cf. [15]), which implied a proof of Fermat's Last Theorem, has been met with approval from the mathematical community and has even aroused great interest in the public (cf. [1], [95]). It connects, in a very fascinating way, different mathematical subjects, such as algebraic geometry and number theory.

The two main mathematical theories involved are those of elliptic curves and of modular forms. The Taniyama-Shimura conjecture relates the numbers of points on elliptic curves over finite fields to Fourier coefficients of certain modular forms of weight two.

An elliptic curve is a special case of a so-called *Calabi–Yau manifold*, namely a Calabi–Yau manifold of dimension one. Calabi–Yau manifolds are of great importance in string theory, a main branch of modern theoretical physics. It is a very natural task to try to extend the results for elliptic curves to Calabi–Yau manifolds of higher dimension. Calabi–Yau manifolds of dimension two are called *K3 surfaces.* Their arithmetic, i.e., their properties over finite fields, has also been studied but we will take one further step forward and concentrate on Calabi–Yau manifolds of dimension three, the so-called *Calabi–Yau threefolds.*

The arithmetic of Calabi–Yau threefolds defined over $\mathbb{Q}$ is mainly determined by the L-series of their middle étale cohomology space. The dimension of this space is a positive even number and can be used to classify Calabi–Yau threefolds. If the dimension is two then the threefold allows no complex deformations and is therefore called *rigid* (and *non-rigid* otherwise). For a rigid Calabi–Yau threefold X that is defined over $\mathbb{Q}$, there is a precise conjecture about its connection with modular forms. There should exist a newform of weight four for some Hecke subgroup $\Gamma_0(N)$, the L-series of which agrees with the L-series of the middle cohomology of X. In this case X is called *modular.*

The conjecture has been checked in several examples previously and there is also a partial general result by Dieulefait and Manoharmayum (a modularity proof under mild restrictions concerning the primes of bad reduction). It is rather difficult to construct rigid Calabi–Yau threefolds.

For non-rigid Calabi–Yau threefolds the situation becomes much more complicated. We expect that the L-series of their middle cohomology is also determined by modular or automorphic forms. There are some examples where the L-series splits into two-dimensional pieces, which are easier to handle.

The main subject of this book is the presentation of known results concerning modularity of Calabi–Yau threefolds and the construction of many new examples.

In chapter 1 we collect the notations and facts concerning Calabi–Yau manifolds and their arithmetic. We also present general modularity results and tools for modularity proofs.

In chapters 2, 3, 4 and 5 we investigate many different examples of Calabi–Yau threefolds and study their modularity. Note that the level of detail is very different for the single examples. A detailed study of all occurring examples would require much more time and space. Nevertheless, the large number of examples makes it possible for the first time to give conjectures about the levels of the occurring newforms. Altogether there are hundreds of new examples of rigid and non-rigid Calabi–Yau threefolds. I would like to accentuate some results:

- In 3.1 and 3.2 the "standard family of quintics" is discussed. We present an equation for the mirror family as a family of quintics. Inside the mirror family there is a rigid Calabi–Yau manifold that corresponds to the Schoen quintic.
- Double coverings of $\mathbb{P}^3$ branched along an octic surface (so-called *double octics*) are investigated in chapter 4. These Calabi–Yau threefolds are easier to handle because their geometry is determined by the (lower-dimensional) branch locus. This leads to large tables of modular examples.
- In 3.2 and 5.1 we construct two rigid Calabi–Yau threefolds with Euler characteristics 32 and 202. To my knowledge these are the smallest (resp. largest) known values. Note that it seems to be possible to produce larger values (cf. 5.11) but this requires additional work.
- The question of which prime numbers can occur in the levels of weight four modular forms connected with Calabi–Yau threefolds is an interesting one. We present examples involving the "new" primes 13, 19, 31 and 37.

In chapter 6 we try to link those modular Calabi–Yau threefolds that have the *same* modular form in their L-series. According to the Tate Conjecture there should be correspondences between them. We present tables of examples and correspondences for examples connected with weight four newforms of small level. Afterwards we discuss the effect of primes of bad reduction on the level and formulate conjectures.

Appendix A contains a table of arrangements of eight planes defined over $\mathbb{Q}$ and the numerical data of the double coverings of $\mathbb{P}^3$ branched along these arrangements.

Appendix B contains tables of modular double coverings of $\mathbb{P}^3$ branched along the union of six planes and a smooth quadric surface.

Appendix C contains tables of weight two and weight four newforms for $\Gamma_0(N)$ with rational coefficients.

To keep the text from further expansion I omitted details on the background in algebraic geometry and number theory. The reader is referred to the standard textbooks of Hartshorne ([47]) on algebraic geometry, Serre ([91]) on Galois representations and Knapp ([58]), Dolgachev ([37]) or Milne ([72]) on modular forms. Further references on specific topics are given in the text. The table of references should be rather complete as far as the subject of modularity of Calabi–Yau threefolds is concerned.

This book is the published version of my thesis, which was written between March 2001 and February 2005 at the Johannes Gutenberg-Universität Mainz, Germany. The modifications concern only the layout but not the contents. It is a pleasure to thank N. Yui for encouraging me to publish my thesis as a book.

I thank everybody who has helped me in one way or another during the time I wrote my thesis. This includes my advisor, D. van Straten, and everybody else working in algebraic geometry at the University of Mainz. The working conditions at the institute of mathematics have been excellent.

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March 1, 2005
Mainz, Germany

CHRISTIAN MEYER

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