



FIELDS INSTITUTE MONOGRAPHS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

Conformal Field Theory with Gauge Symmetry

Kenji Ueno



American Mathematical Society



The Fields Institute
for Research in Mathematical Sciences

FIELDS



FIELDS INSTITUTE MONOGRAPHS

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

Conformal Field Theory with Gauge Symmetry

Kenji Ueno



American Mathematical Society
Providence, Rhode Island



The Fields Institute
for Research in Mathematical Sciences
Toronto, Ontario

FIELDS

The Fields Institute for Research in Mathematical Sciences

The Fields Institute is a center for mathematical research, located in Toronto, Canada. Our mission is to provide a supportive and stimulating environment for mathematics research, innovation and education. The Institute is supported by the Ontario Ministry of Training, Colleges and Universities, the Natural Sciences and Engineering Research Council of Canada, and seven Ontario universities (Carleton, McMaster, Ottawa, Toronto, Waterloo, Western Ontario, and York). In addition there are several affiliated universities and corporate sponsors in both Canada and the United States.

Fields Institute Editorial Board: Carl R. Riehm (Managing Editor), Barbara Lee Keyfitz (Director of the Institute), Juris Steprans (Deputy Director), John Bland (Toronto), Kenneth R. Davidson (Waterloo), Joel Feldman (UBC), R. Mark Goresky (Institute for Advanced Study, Princeton), Cameron Stewart (Waterloo), Noriko Yui (Queen's).

2000 *Mathematics Subject Classification*. Primary 81T40, 81R10, 14D21, 17B81.

For additional information and updates on this book, visit
www.ams.org/bookpages/fim-24

Library of Congress Cataloging-in-Publication Data

Ueno, Kenji, 1945-

Conformal field theory with gauge symmetry / Kenji Ueno.

p. cm. — (Fields institute monographs, ISSN 1069-5273 ; v. 24)

Includes bibliographical references and index.

ISBN 978-0-8218-4088-7 (alk. paper)

1. Conformal invariants. 2. Quantum field theory. 3. Symmetry (Physics) 4. Gauge fields (Physics) I. Title.

QC174.52.C66.U54 2008

530.14'3—dc22

2008022192

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to reprint-permission@ams.org.

© 2008 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

This publication was prepared by the Fields Institute.

<http://www.fields.utoronto.ca>

Visit the AMS home page at <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 13 12 11 10 09 08

Contents

Preface	v
Notation	vii
Chapter 1. Riemann Surfaces and Stable Curves	1
1.1. Compact Riemann surfaces	1
1.2. Stable N -pointed curves	17
1.3. Deformation of pointed curves	25
1.4. Versal family of stable pointed curves	31
Chapter 2. Affine Lie Algebras and Integrable Highest Weight Representations	37
2.1. Affine Lie algebras	37
2.2. Energy-momentum tensor	40
Chapter 3. Conformal Blocks and Correlation Functions	47
3.1. Conformal blocks	47
3.2. Formal neighbourhoods	52
3.3. Basic properties of the space of conformal blocks	54
3.4. Correlation functions	67
Chapter 4. Sheaf of Conformal Blocks	73
4.1. Sheaf of conformal blocks	73
4.2. Local freeness I (smooth case)	76
4.3. Local freeness II (general case) and factorization	82
Chapter 5. Projectively Flat Connections	91
5.1. Projectively flat connections	91
5.2. Sheaf of twisted differential operators	96
5.3. Differential equations near the boundary	101
5.4. Conformal blocks of one-pointed elliptic curves	109
5.5. Verlinde formula	111
5.6. Moduli space of parabolic bundles and Hitchin's connection	116
Chapter 6. Vertex Operators and KZ Equations	131
6.1. Conformal blocks on the Riemann sphere \mathbb{P}^1	131
6.2. KZ equations	136
6.3. Vertex operators	140
6.4. Fundamental solutions of the KZ equations	145
6.5. $A_n^{(1)}$ and Hecke algebra	151
Appendix	161

Bibliography

163

Index

167

Preface

The main purpose of the present notes is to give a systematic approach to conformal field theory with gauge symmetry, the so called the Wess-Zumino-Witten-Novikov model from the viewpoint of complex algebraic geometry. After presenting basic facts on the theory of compact Riemann surfaces, and on the representation theory of affine Lie algebras in Chapters 1 and 2, respectively, we shall construct conformal blocks for stable pointed curves with coordinates in Chapter 3. In Chapter 4 we shall construct the sheaf of conformal blocks associated to a family of stable pointed curves with coordinates. In Chapter 5 it will be shown that the sheaf of conformal blocks carries a projectively flat connection, which is one of the most important facts of conformal field theory. Chapter 6 is devoted to study the detailed structure of the conformal field theory over \mathbb{P}^1 .

Recently J.E. Andersen and I constructed modular functors from conformal field theory. This gives an interesting relationship between Algebraic Geometry and Topological Quantum Field Theory. The present notes include all the necessary techniques and results on conformal field theory with gauge symmetry, which are used to construct the modular functor.

The present notes are based on the lectures and talks given at the Fields Institute, Queen's University, Århus University, Kobe University and Kyoto University. I thank the enthusiastic audiences who helped me to improve certain parts of the proofs in these notes.

My thanks are due to Noriko Yui for inviting me to write the present notes for Fields Institute Monographs and for helping me to complete the manuscript. Last but not least I would like to express my hearty thanks to Arthur Greenspoon of Mathematical Reviews for smoothing out the English of my original manuscript.

March 2008
Kenji Ueno

This page intentionally left blank

Notation

S_ω	the projective connection attached to a symmetric bidifferential ω (see (1.41)),
T_X	the holomorphic (or regular) tangent bundle of a complex manifold (or smooth projective variety) X ,
Θ_X	the sheaf of holomorphic vector fields of a complex manifold or a smooth projective variety X ,
Ω_X^k	the sheaf of holomorphic k -forms of a complex manifold or a smooth projective variety X ,
$\widehat{\mathfrak{g}} = \mathfrak{g} \otimes_{\mathbb{C}} \mathbb{C}((\xi)) \oplus \mathbb{C} \cdot c$,	
$\widehat{\mathfrak{g}}_N = \mathfrak{g} \otimes_{\mathbb{C}} \left(\bigoplus_{j=1}^N \mathbb{C}((\xi_j)) \right) \oplus \mathbb{C} \cdot c$,	
Δ	the root system of a complex simple Lie algebra and its Cartan subalgebra $(\mathfrak{g}, \mathfrak{h})$,
Δ_+	the set of positive roots,
θ	the longest root of a complex simple Lie algebra \mathfrak{g} ,
$\rho = \frac{1}{2} \sum_{\alpha \in \Delta_+} \alpha$,	
$(,)$	Cartan-Killing form of a complex simple Lie algebra \mathfrak{g} normalized as $(\theta, \theta) = 2$,
P_+	the set of integral dominant weights of a complex simple Lie algebra \mathfrak{g} ,
$P_\ell = \{\lambda \in P_+ \mid (\lambda, \theta) \leq \ell\}$,	
$\mu^\dagger = -w(\mu)$	for $\mu \in P_\ell$ where w is the longest element of the Weyl group of a complex simple Lie algebra \mathfrak{g} ,
$\kappa = g^* + \ell$	where g^* is the dual Coxeter number of a complex simple Lie algebra \mathfrak{g} , ($\kappa = n + 1 + \ell$ in case $\mathfrak{g} = \mathfrak{sl}(n + 1, \mathbb{C})$),
$c_v = \frac{\ell \dim \mathfrak{g}}{\kappa}$,	
$\Delta_\lambda = \frac{(\lambda, \lambda + 2\rho)}{2\kappa}$,	
$\widehat{\Delta}_v = \Delta_\lambda + \Delta_{\mu_1} - \Delta_{\mu_2}$,	
$\widehat{\Delta}_v = \Delta_\lambda + \Delta_{\mu_1} - \Delta_{\mu_2}$	for a vertex $\mathbf{v} = \begin{pmatrix} \lambda \\ \mu_1 \mu_2 \end{pmatrix}$,
$X_n = \mathbb{C}^n \setminus \cup_{i < j} \Delta_{ij}$,	$\Delta_{ij} = \{(z_n, \dots, z_1) \in \mathbb{C}^n \mid z_i = z_j\}$,
$\mathcal{R}_n = \{(z_n, \dots, z_1) \in \mathbb{C}^n \mid z_n > z_{n-1} > \dots > z_1 \}$,	

This page intentionally left blank

Appendix

In this appendix we shall give basic results on the hypergeometric function. For any complex number α and for any non-negative integer n define

$$(\alpha)_n = \alpha(\alpha + 1)(\alpha + 2) \cdots (\alpha + n - 1)$$

where

$$(\alpha)_0 = 1.$$

Using the gamma function we can express $(\alpha)_n$ as

$$(\alpha)_n = \frac{\Gamma(\alpha + n)}{\Gamma(\alpha)}$$

The hypergeometric series $F(\alpha, \beta, \gamma; z)$ is defined by

$$F(\alpha, \beta, \gamma; z) = \sum_{n=0}^{\infty} \frac{(\alpha)_n (\beta)_n}{(\gamma)_n n!} z^n = 1 + \frac{\alpha\beta}{\gamma} z + \frac{\alpha(\alpha + 1)\beta(\beta + 1)}{\gamma(\gamma + 1)2!} z^2 + \cdots, \quad (\text{A.39})$$

where α, β, γ are arbitrary complex numbers. The radius of convergence of the series $F(\alpha, \beta, \gamma; z)$ is 1, hence it defines a holomorphic function on the unit disk. The holomorphic function can be analytically extended to a multi-valued holomorphic function on $\mathbb{C} \setminus \{0, 1\}$. We use the same notation $F(\alpha, \beta, \gamma; z)$ for the multi-valued holomorphic function and call it the hypergeometric function. The hypergeometric function $F(\alpha, \beta, \gamma; z)$ is a solution of the hypergeometric differential equation:

$$z(1 - z) \frac{d^2 u}{dz^2} + (\gamma - (\alpha + \beta + 1)z) \frac{du}{dz} - \alpha\beta u = 0. \quad (\text{A.40})$$

The hypergeometric differential equation (A.40) has regular singular points at 0, 1 and ∞ . For simplicity in the following assume that γ is not a negative integer or 0. In a neighbourhood of 0 two linearly independent solutions are given by

$$\begin{aligned} u_{0,1} &= F(\alpha, \beta, \gamma; z) \\ u_{0,2} &= z^{1-\gamma} F(1 - \gamma + \alpha, 1 - \gamma + \beta, 2 - \gamma; z). \end{aligned} \quad (\text{A.41})$$

In a neighbourhood of 1 two linearly independent solutions are given by

$$u_{1,1} = F(\alpha, \beta, \alpha + \beta - \gamma + 1; 1 - z) \quad (\text{A.42})$$

$$u_{1,2} = (1 - z)^{\gamma - \alpha - \beta} F(\gamma - \alpha, \gamma - \beta, \gamma - \alpha - \beta; z). \quad (\text{A.43})$$

Similarly, in a neighbourhood of ∞ two linearly independent solutions are given by

$$u_{\infty,1} = (-z)^{-\alpha} F(\alpha, 1 + \alpha - \gamma, 1 + \alpha - \beta; \frac{1}{z}) \quad (\text{A.44})$$

$$u_{\infty,2} = (-z)^{-\beta} F(\beta, 1 + \beta - \gamma, 1 + \beta - \alpha; \frac{1}{z}). \quad (\text{A.45})$$

The analytic continuation of $F(\alpha, \beta, \gamma; z)$ in a neighbourhood of the point ∞ is given by

$$\begin{aligned}
 F(\alpha, \beta, \gamma; z) &= \frac{\Gamma(\gamma)\Gamma(\beta - \alpha)}{\Gamma(\beta)\Gamma(\gamma - \alpha)} F(\alpha, 1 + \alpha - \gamma, 1 + \alpha - \beta; \frac{1}{z}) \\
 &\quad + \frac{\Gamma(\gamma)\Gamma(\alpha - \beta)}{\Gamma(\alpha)\Gamma(\gamma - \beta)} F(\beta, 1 + \beta - \gamma, 1 + \beta - \alpha; \frac{1}{z}) \quad (\text{A.46})
 \end{aligned}$$

A proof can be found in Whittaker & Watson [WW] 14.5. This fact is used to calculate the connection matrix in Theorem 6.25.

Bibliography

- [A] J. E. Andersen, The Witten invariant of finite order mapping tori I, To appear in *Journal für Reine und Angewandte Mathematik*.
- [AU1] J. E. Andersen & K. Ueno, Abelian conformal field theory and determinant bundles, *Internat. J. Math.* 18, 919–993, 2007.
- [AU2] J. E. Andersen & K. Ueno, Geometric construction of modular functors from conformal field theory. *J. Knot Theory Ramifications* 16, 127–202, 2007.
- [AU3] J. E. Andersen & K. Ueno, Construction of the Reshetikhin-Turaev TQFT from conformal field theory, Preprint in preparation.
- [Ar] M. Artin, Lectures on deformation of singularities *Tata Institute Lecture Notes*, 54, 1976.
- [At] M.F. Atiyah, On framings of 3-manifolds, *Topology* 29 (1990) 1-7
- [BK] B. Bakalov; A. Jr. Kirillov, Lectures on tensor categories and modular functors, *University Lecture Series*, 21. American Mathematical Society, Providence, RI, 2001.
- [Be] A. Beauville, Conformal blocks, fusion rules and the Verlinde formula, *Proceedings of the Hirzebruch 65 Conference on Algebraic Geometry (Ramat Gan, 1993)*, 75–96, Israel Math. Conf. Proc., 9, 1996.
- [BL] A. Beauville and Y. Laszlo, Conformal blocks and generalized theta functions, *Comm. Math. Phys.*, 164, 385–419, 1994.
- [BS] A. A. Beilinson and V. V. Shechtman, Determinant bundles and Virasoro algebras, *Comm. Math. Phys.* 118, 651–701, 1988.
- [BPZ] A. A. Belavin, A. M. Polyakov and A. B. Zamolodchikov, Infinite conformal symmetry in two-dimensional quantum field theory, *Nucl. Phys. B*, 241, 333–380, 1984.
- [BF] D. Bernard and G. Felder, Fock representations and BRST cohomology in $SL(2)$ current algebra, *Comm. Math. Phys.* 127, 145–168, 1990.
- [Bl1] C. Blanchet, Hecke algebras, modular categories and 3-manifolds quantum invariants, *Topology* 39 no. 1, 193–223, 2000.
- [BHMV1] C. Blanchet, N. Habegger, G. Masbaum, P. Vogel, Three-manifold invariants derived from the Kauffman bracket, *Topology* 31 no. 4, 685–699, 1992.
- [BHMV2] C. Blanchet, N. Habegger, G. Masbaum, P. Vogel, Topological quantum field theories derived from the Kauffman bracket, *Topology* 34 no. 4, 883–927, 1995.
- [CL] E. A. Coddington and N. Levinson, Theory of ordinary differential equations *McGraw-Hill* 1955.
- [DM] P. Deligne and D. Mumford, The irreducibility of the space of curves of given genus, *Pub. Math. IHES*, 36, 75–109, 1969.
- [DV] R. Dijkgraaf and E. Verlinde, Modular invariance and the fusion algebra, *Nucl. Phys. B(Proc. Suppl.)*, 5B, 87–97, 1988.
- [EO] T. Eguchi and H. Ooguri, Conformal and current algebras on a general Riemann surface, *Nucl. Phys. B*, 282, 308–328, 1987.
- [Fa] J. D. Fay, Theta functions on Riemann surfaces, *Lecture Notes in Math.*, 352, Springer-Verlag, 1973.
- [Fe] G. Felder, BRST approach to minimal models, *Nucl. Phys. B*, 317, 215–236, 1989.
- [GJ] B. van Geemen and A.J. de Jong, On Hitchin’s connection, *J. AMS*, 11, 189–228, 1998.
- [GHP] L. Gerritzen, F. Herrlich and M. van der Put, Stable n -pointed trees of projective lines, *Nederl. Akad. Wetensch. Indag. Math.*, 50, 131–163, 1988.
- [G] J. Grove, Constructing TQFTs from modular functors, *J. Knot Theory Ramifications* 10 no. 8, 1085–1131, 2001.

- [Ha] R. Hartshorne, Algebraic Geometry, *Springer-Verlag*, 1977.
- [Hi] N. J. Hitchin, Flat connections and geometric quantization, *Comm. Math. Phys.*, 131, 347–380, 1990.
- [Hu] J.E. Humphreys, Introduction to Lie Algebras and Representation Theory, *Springer-Verlag*, 1980.
- [Kac] V. Kac, Infinite dimensional Lie algebras, third edition, *Cambridge University Press*, 1990.
- [Kan] K. Kanie, Conformal field theory and the braid group, *Bulletin Fac. Edu. Mie Univ.* 40, 1–43, (1989).
- [KSU1] T. Katsura, Y. Shimizu and K. Ueno, New bosonization and conformal field theory over \mathbb{Z} , *Comm. Math. Phys.* 121, 603–622, 1988.
- [KSU2] T. Katsura, Y. Shimizu and K. Ueno, Formal groups and conformal field theory over \mathbb{Z} , *Advanced Studies in Pure Mathematics*, 19, 347–366, 1988.
- [KSU3] T. Katsura, Y. Shimizu and K. Ueno, Complex cobordism ring and conformal field theory over \mathbb{Z} , *Math. Ann.* 291, 551–571, 1991.
- [KNTY] N. Kawamoto, Y. Namikawa, A. Tsuchiya and Y. Yamada, Geometric realization of conformal field theory on Riemann surfaces, *Comm. Math. Phys.*, 116(1988), 247–308.
- [KZ] V. G. Knizhnik and A. B. Zamolodchikov, Current algebra and Wess-Zumino model in two dimensions, *Nucl. Phys. B* 247, 83–103, 1984.
- [Kod] K. Kodaira, Complex manifolds and deformation of complex structures, *Springer-Verlag*, 1985.
- [Koh] T. Kohno, Three-manifold invariants derived from conformal field theory and projective representations of modular groups, *Intern. J. Modern Phys.*, 6, 1795–1805, 1992.
- [Kon] M. Kontsevich, Rational conformal field theory and invariants of 3-manifolds, *Preprint of Centre de Physique Theorique Marseille*, CPT-88/p2189, 1988.
- [KNR] S. Kumar, M.S. Narasimhan & A. Ramanan, Infinite Grassmannians and moduli spaces of G -bundles, *Math. Ann.* 300, 395–423, 1994.
- [Ku] S. Kumar, Demazure character formula in arbitrary Kac-Moody setting, *Invent. Math.*, 89, 395–423, 1987.
- [Kur] G. Kuroki, Fock space representations of affine Lie algebras and integral representations in the Wess-Zumino-Witten models, *Comm. Math. Phys.* 142, 511–542, 1991.
- [L] Y. Laszlo, Hitchin's and WZW connections are the same, *J. Differential Geometry*, 49(1998), 547–576.
- [LS] Y. Laszlo and C. Sorger, The line bundles on the stack of parabolic G -bundles over curves and their sections, *preprint*, 1995.
- [Ma] O. Mathue, Formula de caractères pour les algèbres de Kac-Moody généraux, *Astérisque*, 159–160, 1988.
- [MS1] G. Moore and N. Seiberg, Polynomial equations for rational conformal field theories, *Phys. Lett. B*, 212, 451–460, 1988.
- [MS2] G. Moore and N. Seiberg, Classical and quantum conformal field theory, *Comm. Math. Phys.* 123, 177–254, 1989.
- [Se] G. Segal, The definition of conformal field theory, *Topology, geometry and quantum field theory*, 421–577, London Math. Soc. Lecture Note Ser., 308(2004).
- [Serre] J.-P. Serre, Algebraic Groups and Class Fields, Springer, 1988.
- [SGA7] Groupes de monodromie en géométrie algébrique, 1967–68(SGA 7), *Lecture Notes in Math*, No. 288, No.340, 1972, 1973.
(<http://modular.fas.harvard.edu/sga/sga/index.html>).
- [So] C. Sorger, La formule de Verlinde, *Séminaire Bourbaki*, 47ème année, 1994–95, no. 794
- [Su] T. Suzuki, Finite-dimensionality of the space of conformal blocks, *preprint* 1994.
- [T] T. Terada, Quelques propriétés géométriques du domaine de F_1 et le groupe de tresses colorées, *Publ. Res. Inst. Math. Sci.* 17, 95–111, 1981.
- [TK] A. Tsuchiya and Y. Kanie, Vertex operators in conformal field theory on \mathbb{P}^1 and monodromy representations of braid group, *Advanced Studies in Pure Mathematics* 16, 297–326, 1988.
- [TUY] A. Tsuchiya, K. Ueno and Y. Yamada, Conformal field theory on universal family of stable curves with gauge symmetries, *Advanced Studies in Pure Mathematics* 19, 459–566, 1989.

- [Tu] V. Turaev, Quantum invariants of knots and 3-manifolds, *W. de Gruyter*, 1994.
- [U1] K. Ueno, On conformal field theory, *London Math. Soc. Lecture Note* 208, 283–345, 1995.
- [U2] K. Ueno, Introduction to conformal field theory with gauge symmetries, *Geometry and physics (Aarhus, 1995)*, *Lecture Notes in Pure and Appl. Math.* 184, 603–745, Dekker, New York, 1997.
- [V] E. Verlinde, Fusion rules and modular transformations in 2d conformal field theory, *Nucl. Phys. B*, 300 [FS22], 360–376, 1988.
- [Wa] K. Walker, On Witten's 3-manifold invariants, *Preliminary version # 2, Preprint* 1991.
- [Wen] H. Wenzl, Hecke algebra of type A_N and subfactors, *Invent. Math.*, 92, 349–383, 1988.
- [Wit] E. Witten, Quantum field theory and the Jones polynomial, *Comm. Math. Phys.* 121, 351–399, 1989.
- [WW] E.T. Whittaker & G.N. Watson, A Course of Modern Analysis, Fourth Edition, *Cambridge Univ. Press*, 1927.

This page intentionally left blank

Index

- S*-matrix, 111
- ℓC -constraints, 146
- Affine Lie algebra, 38
- Braid group, 156
- Casimir element, 41
- Character
 - integrable highest weight module, 110
- Complex analytic family, 2
- Complex analytic family of elliptic curves, 2
- Composable, 144
- Condition (Q), 18
- Conformal block, 48
- Conformal dimension, 141
- Connection isomorphism, 31
- Correlation function, 68
- Correlation functions
 - of currents, 68
- Covacua, 48
- Critical locus, 29
- Deformation theory, 121
- Discriminant locus, 29
- Dual Coxeter number, 42
- Dualizing sheaf of a nodal curve, 19
- Energy-momentum tensor, 41
- Factorization, 60
- First order deformation, 25
- Formal neighbourhood, 19
- Heat operator, 124
- Hecke algebra, 158
- Highest root, 37
- Highest weight vector, 40
- Hitchin connection, 129
- Infinitesimal deformation, 25
- Initial term, 143
- Integrable highest weight module, 39
- Iwahori-Hecke algebra, 158
- Kanġe basis, 160
- Kodaira-Spencer mapping, 4
- Kodaira-Spencer mapping
 - family of stable pointed curves, 35
- KZ equation, 140
- Level, 39
- Longest root, 37
- Loop algebra, 117
- Loop group, 116
- Monodromy representation, 156
- Nodal curve, 17
- Normal ordering, 40
- Normalized Cartan-Killing form, 37
- Parabolic G -bundle, 118
- Parabolic subgroup, 116
- Projective connection, 17
- Projective flat connection, 93
- Projective heat operator, 124
- Propagation of vacua, 54
- Quasi-parabolic G -bundle, 118
- Residue pairing, 20
- Restricted braid group, 156
- Riemann surface, 1
- Root vector, 37
- Schwarzian derivative, 53
- Sewing, 84
- Sheaf of covacua, 74
- Sheaf of twisted differential operators, 96
- Sheaf of conformal blocks, 74
- Sheaf of vacua, 74
- Space of covacua, 48
- Space of vacua, 48
- Universal, 27
- Universal moduli stack of G -bundles, 121
- Vacua, 48
- Verlinde algebra, 112
- Verma module, 39
- Versal, 27
- Versal family, 28, 77
- Vertex, 141
- Vertex operator, 141
- Virasoro algebra, 43

Virasoro operator, 41

Wenzl's representation, 159

Young diagram, 151

This book presents a systematic approach to conformal field theory with gauge symmetry from the point of view of complex algebraic geometry. After presenting the basic facts of the theory of compact Riemann surfaces and the representation theory of affine Lie algebras in Chapters 1 and 2, conformal blocks for pointed Riemann surfaces with coordinates are constructed in Chapter 3. In Chapter 4 the sheaf of conformal blocks associated to a family of pointed Riemann surfaces with coordinates is constructed, and in Chapter 5 it is shown that this sheaf supports a projective flat connection—one of the most important facts of conformal field theory. Chapter 6 is devoted to the study of the detailed structure of the conformal field theory over \mathbb{P}^1 .

Recently it was shown that modular functors can be constructed from conformal field theory, giving an interesting relationship between algebraic geometry and topological quantum field theory. This book provides a timely introduction to an intensively studied topic of conformal field theory with gauge symmetry by a leading algebraic geometer, and includes all the necessary techniques and results that are used to construct the modular functor.

ISBN 978-0-8218-4088-7



9 780821 840887



For additional information
and updates on this book, visit

www.ams.org/bookpages/fim-24

FIM/24

AMS on the Web
www.ams.org